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DYNAMIC AND NONLINEAR PROGRAMMING FOR OPTIMUM FARM PLANS IN TAIWAN

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I. INTRODUCTION

Taiwan lies in the sub-tropical region with high temperatures, strong sunlight and abundant rainfall. It is suitable for the production of rice, sweet potatoes, sugar cane and other crops. Crops can be grown on the farms throughout the year, due to the favorable natural conditions. According to the data of 1958 (21, p. 30), rice is the most important crop, followed by sweet potatoes, sugar cane, peanuts, tobacco, soybeans, tea, and bananas. The value of rice production is approximately 41.43% of the total value of agricultural products with 9.13%, 7.90% and 3.30% for sweet potatoes, sugar cane and peanuts, respectively.

Since the land resource is limited in Taiwan, keen competition among crops has existed in the use of land. For example, spring planted crops such as spring sweet potatoes, spring "hu-tze"[1] sweet potatoes, spring peanut and jute are grown on the same field of single-cropping paddy land at the same time. Since the farm size is very small, usually only one crop is produced. Hence these crops compete directly for use of the land.

Most farmers in Taiwan do not allocate their resources

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[1]"Hu-tze" is an interplanting method wherein a crop is planted in the field a few weeks before the harvest of the previous crop. For example, spring planted "hu-tze" sweet potatoes are planted in late October before the harvest of the second rice crop.

properly and, as a result, they do not maximize their profits. Several studies (4, 5, 6) have been done by this author on optimum farm plans for single-crop paddy farms in Taiwan from the standpoint of resource allocation and crop combination under the following situations: (a) resources and prices fixed; (b) one resource variable; (c) two resources variable; (d) one price variable; (e) two prices variable. These conventional linear programs are based on a simple linear input-output relationship and are only for a 1-year period. The former may deviate from the "real world" of farm production, and the latter does not meet the need of farm operators.

In this study, a case farm is selected for a five-year dynamic linear programming study. The optimum farm and home plans for dynamic linear programming are for a period of 5 years, where the plan for each year is the most profitable one. The separation method is developed and other recent techniques are applied for solving the 5-year dynamic linear program for the case farm. Optimum plans are presented for the case farm based on the following situations: (a) with land and labor fixed; (b) with the hiring of spring labor; (c) with the renting of land; and (d) with the hiring of spring labor and the renting of land.

Since the case farm operates under conditions of increasing marginal productivity for spring labor, the nonlinear programming approach is applied for determination of the optimum plan of the case farm. The modified simplex method and the mixed integer programming algorithm are used to solve the nonlinear problem of the case farm successfully. The former is developed by this study and is easy to apply.

This study attempts to demonstrate how to apply dynamic and nonlinear programming approaches to solve farm and home management problems for Taiwan. It also provides guidance information to help farmers to allocate their resources between farm and home properly, over a period of years, as well as within increasing return stages, in order to maximize their profits.

II. OBJECTIVES OF STUDY

The central objective of this study is to develop and apply methods of dynamic and nonlinear programming for determining optimum farm and home plans on a single crop paddy farm in Chaiyi, Taiwan. The more specific objectives are:

1. To develop new and simplified methods to solve dynamic and nonlinear programming problems for the case farm.
2. To apply recent developments in programming techniques, both dynamic and nonlinear, to case farm problems.
3. To determine optimum farm and home plans for the case farm over a period of five years given the restraints of available resources and family living costs.
4. To determine optimum 5-year farm and home plans at different levels of labor and land supplies for the case farm.
5. To determine optimum farm and home plans for the case farm under the situation of increasing marginal productivity.
6. To show how the different methods of dynamic and nonlinear programming can be used to solve case farm problems.
7. To demonstrate how dynamic and nonlinear programming can be used on the study of farm and home management

problems in Taiwan.

8. To provide basic information for guidance in farm organization and home management to farmers in Taiwan for the purpose of increased profits for farms.

III. CASE FARM SITUATIONS

A. Description of the Case Farm

1. Resources

The case farm selected for dynamic and nonlinear programming is in Chaiyi, Taiwan. The farm is owner-operated. Its land is classified as single-cropping paddy land. This indicates that rice can be planted on the land only once a year. The currently available resources of the farm are: 2 hectares of land, NT$17,000[1] of operating capital, 190 days of labor (utilized in the spring and in the fall). The labor is provided by the operator and his family. The amount of available operating capital and labor will depend upon production and consumption in the preceding year, and the operator's son growing up to supply more labor. The operator, consequently, expects the amount of capital and labor to vary from year to year. Average management is assumed for crop production on the case farm. Predicted farm resources for 5 years are presented in Table 1.

2. Crop enterprises

It is possible for the case farm to grow within the year the following three groups of crops:

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[1]The rate of exchange is 1 United States dollar to 36 new Taiwan dollars.

Table 1. Expected resources for the case farm

Year	Spring land	Fall land	Operating capital[a]	Spring labor	Fall labor
	ares[b]	ares	NT$	days	days
1	200	200	17,000	190	190
2	200	200	--	195	195
3	200	200	--	215	215
4	200	200	--	220	220
5	200	200	--	225	225

[a]In years 2, 3, 4 and 5, the amount of available operating capital depends upon production and consumption in the preceding year.

[b]"Are" is a unit of surface measure in the metric system, equal to 100 square meters or 0.01 hectare.

a. Spring planted crops: spring sweet potatoes, spring "hu-tze" sweet potatoes, spring peanut and jute.

b. Fall planted crops: fall sweet potatoes, second rice and fall peanut.

c. Annually planted crop: ratoon sugar cane.

3. Input-output coefficients

Input-output coefficients are required for the crops grown on the case farm.

As a step in establishing input-output coefficients, it is necessary to establish labor and capital requirements or inputs per are. In linear programming, these inputs are taken to be constants per are of land. We use the price index in the previous 5 years and the operator's prediction to project the costs during the 5-year period. These costs increase each year. It is assumed that there are no technological improvements in agriculture during the planning period. Consequently, the labor requirement per are for the crop is the same for the 5 years. The operating capital and labor requirements per are for the several crops for the 5 years are shown in Table 2.

4. Prices and yields

The prices of each crop for the 5 years are also projected on the basis of the price index of the previous 5 years and the operator's expectations of prices. The yield for each crop is the average yield for the previous 5 years on the case farm or on neighboring farms.[1] The prices and yields are shown in Table 3.

5. Discounted net revenues

The prices in Table 3 are used to compute net revenue. Net revenue is the yield per are of an activity times the

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[1]As some crops were not grown on the case farm during the previous 5 years, it is necessary to use the average yield of these crops from the neighboring farms.

Table 2. Projected capital and labor requirements per are for the case farm

Crop enterprise		Operating capital years number 1	2	3	4	5
Spring sweet potatoes	P_1	20.67970	21.92048	23.23570	24.62984	26.10763
Spring "hu-tze" sweet potatoes	P_2	21.94210	23.25862	24.65413	26.13337	27.70137
Spring peanut	P_3	57.26480	60.70069	64.34273	68.20329	72.29548
Jute	P_4	94.07904	98.78299	103.72214	108.90825	114.35366
Fall sweet potatoes	P_5	79.34400	84.89808	90.84094	97.19980	104.00378
Second rice	P_6	81.69768	85.78256	90.07169	94.57527	99.30403
Fall peanut	P_7	69.98940	74.18876	78.64008	83.35848	88.35999
Ratoon sugar cane	P_8	127.12504	136.02379	145.54545	155.73363	166.63498

ase farm

5	Spring labor (from years 1 through 5)	Fall labor (from years 1 through 5	Spring land	Fall land
26.10763	0.38970	0	1	0
27.70137	0.51480	0	1	0
72.29548	0.76820	0	1	0
114.35366	1.33860	0	1	0
104.00378	0	0.32520	0	1
99.30403	0	0.59840	0	1
88.35999	0	0.68590	0	1
166.63498	0.96420	0.37990	1	1

Table 3. Projected yield and price of product for the case farm

Crop		Yield per are from years 1 through 5 (Kg)	Price per kilogram (NT$) Year 1	2	3	4	5
Spring sweet potatoes	P_1	174.00010	0.37	0.40	0.42	0.45	0.48
Spring "hu-tze" sweet potatoes	P_2	217.06420	0.33	0.35	0.38	0.40	0.43
Spring peanut	P_3	11.43010	5.01	5.31	5.63	5.97	6.33
Jute	P_4	20.40760	4.61	4.84	5.08	5.34	5.60
Fall sweet potatoes	P_5	165.30000	0.48	0.51	0.55	0.59	0.63
Second rice	P_6	37.47600	2.18	2.29	2.40	2.52	2.65
Fall peanut	P_7	13.45950	5.20	5.51	5.84	6.19	6.56
Ratoon sugar cane	P_8	794.53150	0.16	0.17	0.18	0.20	0.21

price per kilogram of the activity less the cost of operating capital. Since this study is designed for a 5-year dynamic linear program, the discounted net revenue must be used for programming. At a 9 per cent interest rate, the discounted net revenues are presented in Table 4.

Table 4. Discounted net revenue per are for the case farm (NT$)

Year	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	Spring peanut P_3	Jute P_4	Fall sweet potatoes P_5	Second rice P_6	Fall peanut P_7	Ratoon sugar cane P_8
1	43.70	49.69	36.13	61.46	63.09	67.87	50.49	94.66
2	43.09	48.98	35.14	58.91	62.09	65.25	49.10	93.22
3	42.48	48.28	34.17	56.45	61.09	62.73	47.75	91.80
4	41.88	47.58	33.23	54.10	60.11	60.31	46.43	90.40
5	41.29	46.89	32.16	51.84	59.15	57.98	45.16	89.02

6. Fixed farm expenditures

The fixed farm expenditures of the case farm include expenses for building, added investment, new implements, interest on investment, taxes and water rent. As this farm is owned by the operator, the interest on investment is about eight-tenths of the total fixed expenditures. The projected fixed expenditures are shown in Table 5.

Table 5. Projected fixed farm expenditures for the case farm (NT$)

Item	Year 1	2	3	4	5
Expenses for building	840	840	900	900	900
Added investment and new implements	300	400	500	550	580
Interest on investment	8,200	8,250	8,300	8,310	8,320
Taxes	840	840	880	900	920
Water rent	410	410	440	440	440
Total	10,590	10,740	11,020	11,100	11,160

7. Family living costs

The family living costs of the case farm are categorized into seven items for each year. These items are carefully estimated by the farm family. The family projected planned expenditures for such items as their son's education fees, a sewing machine, new clothes, a radio receiving set, new furniture, etc. The projected living costs are shown in Table 6.

Table 6. Projected family living costs for the case farm (NT$)

Item	Year				
	1	2	3	4	5
Food	6,100	6,200	6,500	6,700	7,000
Clothes	936	936	1,000	1,050	1,100
Repair and furnishings	500	500	550	600	650
Health	300	350	400	450	500
Education	700	720	1,750	1,800	1,850
Principal ceremonies	900	900	920	950	950
Others	1,050	1,100	1,150	1,200	1,250
Total	10,486	10,706	12,270	12,750	13,300

B. Farm Programming Situation

A 5-year dynamic linear program for the case farm will be carried out under the following situations:

Situation I: The case farm with land and labor fixed as shown in Table 1.

Situation II: Situation I with the hiring of spring labor.

Situation III: Situation I with the renting of land (not more than 1 hectare from years 3 through 5).

Situation IV: Situation I with the hiring of spring labor and the renting of land (not more than 1 hectare from years 3 through 5).

IV. DYNAMIC LINEAR PROGRAMMING FOR THE CASE FARM

Dynamic linear programming represents a general technique for solving multi-stage and multi-activity decision process problems. If the time span is divided into periods, the initial resources provide the inputs for the activities in the first stage. The output from the first stage supplies the resource input for activities in the second stage, etc. Hence, activities in each of the t-years (where t is a finite number) are inter-related. The optimum plans for dynamic linear programming are for a period of t-years, where the plan for each year is the most profitable one.

A. The Modified Simplex Method

1. Logic and technique

A dynamic linear programming model can be developed by modifying the simplex method. Returns from the activities will be maximized for k discrete periods of years ($k = 1, \ldots, t$), subject to restrictions in the availability of resources in all the k years.

In a dynamic model one identifies each coefficient with a particular time period in the Hicksian sense. The year of the program is denoted by the subscript k, where $k = 1, 2, \ldots, t$; the number of the row (or restriction) by i, where $i = 1, 2, \ldots, m$; and the number of the column (or activity) by j, where $j = 1, 2, \ldots, n$. The input-output coefficient a_{ijk} represents that the amount of the i-th resource used per unit of the j-th activity in the k-th year. Activity level x_{jk} represents the level of the j-th activity in the k-th year. Resource supply or restraints b_{ik} represent the i-th resource supply in the k-th year. The net revenue c_{jk} represents the net revenues of the j-th activity in the k-th year. With these notations, the dynamic linear programming model can be expressed in the following equations:

$$
\begin{aligned}
(1)\quad & b_{11} \geq a_{111}\, x_{11} + a_{121}\, x_{21} + \cdots + a_{1j1}\, x_{j1} + a_{1j2}\, x_{j2} + \\
& \qquad \cdots + a_{1nt}\, x_{nt} \\
& b_{21} \geq a_{211}\, x_{11} + a_{221}\, x_{21} + \cdots + a_{2j1}\, x_{j1} + a_{2j2}\, x_{j2} + \\
& \qquad \cdots + a_{2nt}\, x_{nt} \\
& \vdots \\
& b_{i1} \geq a_{i11}\, x_{11} + a_{i21}\, x_{i1} + \cdots + a_{ij1}\, x_{j1} + a_{ij2}\, x_{j2} + \\
& \qquad \cdots + a_{int}\, x_{nt} \\
& b_{i2} \geq a_{i12}\, x_{12} + a_{i22}\, x_{12} + \cdots + a_{ij2}\, x_{j2} + a_{ij2}\, x_{j2} + \\
& \qquad \cdots + a_{int}\, x_{nt} \\
& \vdots \\
& b_{ik} \geq a_{i1k}\, x_{1k} + a_{i2k}\, x_{2k} + \cdots + a_{ijk}\, x_{jk} + a_{ijk}\, x_{jk} + \\
& \qquad \cdots + a_{int}\, x_{nt}
\end{aligned}
$$

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$$b_{mt} \geq a_{m1t}\, x_{1t} + a_{m2t}\, x_{2t} + \cdots + a_{mjt}\, x_{jt} + a_{mjt}\, x_{jt} + \cdots + a_{mnt}\, x_{nt}$$

The objective is to maximize:

(2) $f(X) = c_{11}\, x_{11} + c_{21}\, x_{21} + \cdots + c_{jk}\, x_{jk} + \cdots + c_{nt}\, x_{nt}$

where c_{jk} is discounted net price of j-th activity in k-th year subject to:

(3) $$X_{jk} \geq 0 \quad .$$

"Slack" or "disposal" activities are added, and the inequalities of equations 1 become the equalities in equation 4 below. In general, the number of disposal activities will equal the number of restrictions. With m resource restrictions and n real activities, the total number of activities is $m + n = r$, and j now has the range $j = 1, 2, \ldots, r$. The activity x_{jk} $(j = n + 1, n + 2, \ldots, n + m)$ is a "disposal" activity. The input-output coefficients, corresponding to the "disposal" activities, are in the form:

$a_{ijk} = 1$ $(i = 1, 2, \ldots, m$, and $j = n + 1, n + 2, \ldots, n + m)$

where $i = j - n$, and

$a_{ijk} = 0$

where $i \neq j - n$

which is an identity matrix.

With disposal activities added, it is possible and necessary to have a program with resource requirements exactly equal to supplies as in equation 4.

$$(4) \quad b_{11} = a_{111}\, x_{11} + a_{121}\, x_{21} + \cdots + a_{1j1}\, x_{j1} + \cdots + a_{1nt}\, x_{nt} + a_{1(n+1)1}\, x_{(n+1)1} + \cdots + a_{1rt}\, x_{rt}$$

$$b_{21} = a_{211}\, x_{11} + a_{221}\, x_{21} + \cdots + a_{2j1}\, x_{j1} + \cdots + a_{2nt}\, x_{nt} + a_{2(n+1)1}\, x_{(n+1)1} + \cdots + a_{2rt}\, x_{rt}$$

$$\vdots$$

$$b_{i1} = a_{i11}\, x_{11} + a_{i21}\, x_{i1} + \cdots + a_{ij1}\, x_{j1} + \cdots + a_{int}\, x_{nt} + a_{i(n+1)1}\, x_{(n+1)1} + \cdots + a_{irt}\, x_{rt}$$

$$b_{i2} = a_{i12}\, x_{12} + a_{i22}\, x_{i2} + \cdots + a_{ij2}\, x_{j2} + \cdots + a_{int}\, x_{nt} + a_{i(n+1)1}\, x_{(n+1)1} + a_{irt}\, x_{rt}$$

$$\vdots$$

$$b_{ik} = a_{i1k}\, x_{1k} + a_{i2k}\, x_{2k} + \cdots + a_{ijk}\, x_{jk} + \cdots + a_{int}\, x_{nt} + a_{i(n+1)k}\, x_{(n+1)k} + \cdots + a_{irt}\, x_{rt}$$

$$b_{mt} = a_{m1t}\, x_{1t} + a_{m2t}\, x_{2t} + \cdots + a_{mjt}\, x_{jt} + \cdots + a_{mnt}\, x_{nt} + a_{m(n+1)t}\, x_{(n+1)t} + \cdots + a_{mrt}\, x_{rt}$$

With dynamic linear programming, there is also inter-year resource restrictions. If it be assumed that the operating capital is an inter-year resource restriction, then any activity produced in the k-th year has a positive coefficient in the operating capital equation for year k + 1. For example, activity x_{1k} may require operating capital a_{11k}, say \$10, and yield a net return c_{1k}, say \$30. A unit of this activity produced in year k will add $a_{11k} + c_{1k}$ = \$10 + \$30 = \$40 to the supply of capital in year k + 1. As \$40 is added to the capital supply of the next year, -40 becomes the coefficient in the column x_{1k} and the row for capital supply in year k + 1.

If we take b_{11} as the capital supply in year 1, where $b_{21} \ldots b_{i1}$ represent other resource restrictions, then the remaining equations b_{1k}* of operating capital supply for year 2 through t - 1 are enlarged, due to the operating capital transfer process. Since year t is the final year, there are no resource transfer activities required. Equation 4 will become:

$$(5) \quad b_{11} = a_{111}\,x_{11} + a_{121}\,x_{21} + \cdots + a_{1j1}\,x_{j1} + \cdots + a_{1nt}\,x_{nt} + a_{1(n+1)1}\,x_{(n+1)1} + \cdots + a_{1rt}\,x_{rt}$$

$$b_{21} = a_{211}\,x_{11} + a_{221}\,x_{21} + \cdots + a_{2j1}\,x_{j1} + \cdots + a_{2nt}\,x_{nt} + a_{2(n+1)1}\,x_{(n+1)1} + \cdots + a_{2rt}\,x_{rt}$$

$$\vdots$$

$$b_{i1} = a_{i11}\,x_{11} + a_{i21}\,x_{i1} + \cdots + a_{ij1}\,x_{j1} + \cdots + a_{int}\,x_{nt} + a_{i(n+1)1}\,x_{(n+1)1} + \cdots + a_{irt}\,x_{rt}$$

$$b^*_{i2} = -a_{i1(2-1)}\, x_{1(2-1)} - a_{i2(2-1)} - \cdots -$$
$$a_{ij(2-1)}\, x_{j(2-1)} - \cdots - a_{in(2-1)}\, x_{n(2-1)} +$$
$$a_{i12}\, x_{12} + a_{i22}\, x_{22} + \cdots + a_{ij2}\, x_{j2} + \cdots +$$
$$a_{int}\, x_{nt} + a_{i(n+1)2}\, x_{(n+1)2} + \cdots + a_{irt}\, x_{rt}$$

$$b_{i2} = a_{i12}\, x_{12} + a_{i22}\, x_{22} + \cdots + a_{ij2}\, x_{j2} + \cdots +$$
$$a_{int}\, x_{nt} + a_{i(n+1)2}\, x_{(n+1)2} + \cdots + a_{irt}\, x_{rt}$$

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$$b^*_{ik} = -a_{i1(k-1)}\, x_{1(k-1)} - a_{i2(k-1)}\, x_{2(k-1)} - \cdots -$$
$$a_{ij(k-1)}\, x_{j(k-1)} - \cdots - a_{in(k-1)}\, x_{n(k-1)} +$$
$$a_{i1k}\, x_{1k} + a_{i2k}\, x_{2k} + \cdots + a_{ijk}\, x_{jk} + \cdots +$$
$$a_{int}\, x_{nt} + a_{i(n+1)k}\, x_{(n+1)k} + \cdots + a_{irt}\, x_{rt}$$

$$b_{ik} = a_{i1k}\, x_{1k} + a_{i2k}\, x_{2k} + \cdots + a_{ijk}\, x_{jk} + \cdots +$$
$$a_{int}\, x_{nt} + a_{i(n+1)k}\, x_{(n+1)k} + \cdots + a_{irt}\, x_{rt}$$

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$$b^*_{i(t-1)} = -a_{i1(t-1-1)}\, x_{1(t-1-1)} - a_{i2(t-1-1)}\, x_{2(t-1-1)} -$$
$$\cdots - a_{ij(t-1-1)}\, x_{j(t-1-1)} - \cdots -$$
$$a_{in(t-1-1)}\, x_{n(t-1-1)} + a_{i1(t-1)}\, x_{1(t-1)} +$$
$$a_{i2(t-1)}\, x_{2(t-1)} + \cdots + a_{ij(t-1)}\, x_{j(t-1)} +$$
$$\cdots + a_{int}\, x_{nt} + a_{i(n+1)(t-1)}\, x_{(n+1)(t-1)} +$$

$$\cdots + a_{irt}\, x_{rt}$$

$$b_{i(t-1)} = a_{i1(t-1)}\, x_{1(t-1)} + a_{i2(t-1)}\, x_{2(t-1)} + \cdots +$$
$$a_{ij(t-1)}\, x_{j(t-1)} + \cdots + a_{int}\, x_{nt} +$$
$$a_{i(n+1)(t-1)}\, x_{(n+1)(t-1)} + \cdots + a_{irt}\, x_{rt}$$

$$\vdots$$

$$b_{mt} = a_{m1t}\, x_{1t} + a_{m2t}\, x_{2t} + \cdots + a_{mjt}\, x_{jt} + \cdots +$$
$$a_{mnt}\, x_{nt} + a_{m(n+1)t}\, x_{(n+1)t} + \cdots + a_{mrt}\, x_{rt}$$

where

(6) $x_{jk} \geq 0$, and we maximize:

(7) $$f(X) = \sum_{j=1}^{r} c_{jk}\, x_{jk}\ , \qquad k = 1, 2, \ldots, t$$

where c_{jk} is the discounted value of $\bar{c}_{jk}$, which is the net price of the j-th activity in the k-th year, and can be defined as:

$$c_{jk} = \frac{\bar{c}_{jk}}{(1+r)^k}$$

where r is the market interest rate.

The above dynamic linear programming problem can now be expressed formally and compactly in matrix form as follows:

(8) Maximize: $f(X) = C'\, X$

(9) Subject to: $A\, X = B$, and

(10) $X \geq 0$

f(X) is the objective function which is to be maximized. C' denotes the transpose of C and is a matrix of the present discounted value c_{jk} in equation 7; the x_{jk} values from equation 5 form a column vector, X, of level activities. Hence equation 8 is simply a restatement of equation 7, where the abbreviations for the matrices are included rather than their elements. The a_{ijk} coefficients in equation 5 form a matrix, A, of input-output coefficients; column vector X indicates the level of activities of real, disposal or inter-year transfer. The b_{ik} quantities in equation 5 form a column vector, B, of resource supplies; so that AX provides scalar quantities which are all equal to the corresponding ones in vector B of resource supplies. Hence, equation 9 is also a restatement of equation 5. Equation 10 states that the quantity of each activity level, x_{jk}, contained in X must be equal to or greater than zero. It is the same as equation 6.

The logic and procedure of the modified simplex method to solve a dynamic linear programming problem are also explained and illustrated in several sources (19, 22).

2. Analysis of results and optimum plans

The dynamic linear programming solutions for the case farm were computed with an IBM-650 Magnetic Drum Processing Machine. The modified simplex method as indicated in the previous section was used. The data provided in Tables 1, 2, 4, 5 and 6 of the previous section are sufficient to make a dynamic linear programming solution for maximum profit of the case farm under the following different situations.

a. <u>Situation I: Optimum 5-year plan for the case farm with land and labor fixed</u> The original input-output matrix for a 5-year dynamic linear program is presented in Table 7.

It is desired to "force in" the family living costs activity in the optimum plan each year. Therefore, a larger M value is placed above the activity in Table 7. z_{jk} is the opportunity cost of the j-th activity in year k. Given that c_{jk} is the discounted net revenue of the j-th activity in year k, it then follows that $z_{jk} - c_{jk}$ is the marginal revenue of the j-th activity in year k. For the purpose of simplifying the presentation, the disposal activities, which form an identity matrix, follow the real activities.

An optimum 5-year plan for the case farm, with land and labor fixed, is shown in Table 8.

<u>In year 1</u>, after the deduction of family living costs from operating capital is made, there is only NT$6,514 available for crop production in year 1. The highest return from operating capital is second rice. Accordingly, operating capital is first used for second rice production, and secondly for spring "hu-tze" sweet potatoes which give the next highest return to the operating capital. All 200 ares of fall land are utilized for second rice production. It is due to the

Table 7. Original input-output matrix for a 5-year dynamic linear programming

Year	Resource or activity		Resource or activity level P_0	Year 1: Spring sweet potatoes$_1$ P_1	Spring "hu-tze" sweet potatoes$_1$ P_2	Spring peanut$_1$ P_3	Jute$_1$ P_4	Fall sweet potatoes$_1$ P_5	
1	Spring land$_1$	P_{47}	200 ares	1	1	1	1	0	0
1	Fall land$_1$	P_{48}	200 ares	0	0	0	0	1	1
1	Operating capital$_1$	P_{49}	17,000 NT$	20.67970	21.94210	21.13290	32.61800	16.24990	13.
1	Spring labor$_1$	P_{50}	190 days	0.38970	0.51480	0.76820	1.33860	0	0
1	Fall labor$_1$	P_{51}	190 days	0	0	0	0	0.32520	0.
1	Family living$_1$	P_{52}	10,486 NT$	0	0	0	0	0	0
2	Spring land$_2$	P_{53}	200 ares						
2	Fall land$_2$	P_{54}	200 ares						
2	Operating capital$_2$	P_{55}	0	-64.37970	-71.63210	-57.26290	-94.07800	-79.33990	-81.
2	Spring labor$_2$	P_{56}	195 days						
2	Fall labor$_2$	P_{57}	195 days						
2	Family living$_2$	P_{58}	10,706 NT$						
3	Spring land$_3$	P_{59}	200 ares						
3	Fall land$_3$	P_{60}	200 ares						
3	Operating capital$_3$	P_{61}	0						
3	Spring labor$_3$	P_{62}	215 days						
3	Fall labor$_3$	P_{63}	215 days						
3	Family living$_3$	P_{64}	12,270 NT$						
4	Spring land$_4$	P_{65}	200 ares						
4	Fall land$_4$	P_{66}	200 ares						
4	Operating capital$_4$	P_{67}	0						
4	Spring labor$_4$	P_{68}	220 days						
4	Fall labor$_4$	P_{69}	220 days						
4	Family living$_4$	P_{70}	12,750 NT$						
5	Spring land$_5$	P_{71}	200 ares						
5	Fall land$_5$	P_{72}	200 ares						
5	Operating capital$_5$	P_{73}	0						
5	Spring labor$_5$	P_{74}	225 days						
5	Fall labor$_5$	P_{75}	225 days						
5	Family living$_5$	P_{76}	13,300 NT$						
	Z_{jk}		0	0	0	0	0	0	0
	$Z_{jk} - C_{jk}$		0	-43.70	-49.69	-36.13	-61.46	-63.09	-67.8

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ll et toes$_1$	2nd rice$_1$ P_6	Fall peanut$_1$ P_7	Ratoon sugar cane$_1$ P_8	Family living$_1$ P_9	Spring sweet potatoes$_2$ P_{10}	Spring "hu-tze" sweet potatoes$_2$ P_{11}	Spring peanut$_2$ P_{12}	Jute P_{13}
	0	0	1	0				
	1	1	1	0				
4990	13.82870	19.50130	32.46140	1				
	0	0	0.96420	0				
2520	0.59840	0.68590	0.37990	0				
	0	0	0	1				
					1	1	1	1
					0	0	0	0
3990	-81.69870	-69.99130	-127.12140	0	21.92048	23.25862	22.40087	34.57
					0.38970	0.51480	0.76820	1.33
					0	0	0	0
					0	0	0	0
					-65.01048	-72.23862	-57.54087	-93.48
	0	0	0	0	0	0	0	0
	-67.87	-50.49	-94.66	-M	-43.03	-48.98	-35.14	-58.91

Year 2						
Spring eanut$_2$ P_{12}	Jute$_2$ P_{13}	Fall sweet potatoes$_2$ P_{14}	2nd rice$_2$ P_{15}	Fall peanut$_2$ P_{16}	Ratoon sugar cane$_2$ P_{17}	Family living$_2$ P_{18}
1	1	0	0	0	1	0
0	0	1	1	1	1	0
2.40087	34.57508	17.22489	14.65842	20.67138	34.40908	1
0.76820	1.33860	0	0	0	0.96420	0
0	0	0.32520	0.59840	0.68590	0.37990	0
0	0	0	0	0	0	1
7.54087	-93.48508	-79.31489	-79.90842	-69.77138	-127.62908	0
	0	0	0	0	0	0
.14	-58.91	-62.09	-65.25	-49.10	-93.22	-M

Table 7. (Continued)

Year	Resource or activity		Resource or activity level P_0	Year 3: Spring sweet potatoes$_3$ P_{19}	Spring "hu-tze" sweet potatoes$_3$ P_{20}	Spring peanut$_3$ P_{21}	Jute$_3$ P_{22}	Fall sweet potatoes$_3$ P_{23}
1	Spring land$_1$	P_{47}	200 ares					
1	Fall land$_1$	P_{48}	200 ares					
1	Operating capital$_1$	P_{49}	17,000 NT$					
1	Spring labor$_1$	P_{50}	190 days					
1	Spring labor$_1$	P_{51}	190 days					
1	Family living$_1$	P_{52}	10,486 NT$					
2	Spring land$_2$	P_{53}	200 ares					
2	Fall land$_2$	P_{54}	200 ares					
2	Operating capital$_2$	P_{55}	0					
2	Spring labor$_2$	P_{56}	195 days					
2	Fall labor$_2$	P_{57}	195 days					
2	Family living$_2$	P_{58}	10,706 NT$					
3	Spring land$_3$	P_{59}	200 ares	1	1	1	1	0
3	Fall land$_3$	P_{60}	200 ares	0	0	0	0	1
3	Operating capital$_3$	P_{61}	0	23.23570	24.65413	23.74492	36.64958	18.25838
3	Spring labor$_3$	P_{62}	215 days	0.38970	0.51480	0.76820	1.33860	0
3	Fall labor$_3$	P_{63}	215 days	0	0	0	0	0.32520
3	Family living$_3$	P_{64}	12,270 NT$	0	0	0	0	0
4	Spring land$_4$	P_{65}	200 ares					
4	Fall land$_4$	P_{66}	200 ares					
4	Operating capital$_4$	P_{67}	0	-65.71570	-72.93413	-57.91492	-93.09958	-79.34838
4	Spring labor$_4$	P_{68}	220 days					
4	Fall labor$_4$	P_{69}	220 days					
4	Family living$_4$	P_{70}	12,750 NT$					
5	Spring land$_5$	P_{71}	200 ares					
5	Fall land$_5$	P_{72}	200 ares					
5	Operating capital$_5$	P_{73}	0					
5	Spring labor$_5$	P_{74}	225 days					
5	Fall labor$_5$	P_{75}	225 days					
5	Family living$_5$	P_{76}	13,300 NT$					
	Z_{jk}		0	0	0	0	0	0
	$Z_{jk} - C_{jk}$		0	-42.48	-48.28	-34.17	-56.45	-61.09

Year 3									Ye
Fall sweet potatoes$_3$ P_{23}	2nd rice$_3$ P_{24}	Fall peanut$_3$ P_{25}	Ratoon sugar cane$_3$ P_{26}	Family living$_3$ P_{27}	Spring sweet potatoes$_4$ P_{28}	Spring "hu-tze" sweet potatoes$_4$ P_{29}	Spring peanut$_4$ P_{30}	Jute$_4$ P_{31}	s po
0	0	0	1	0					
1	1	1	1	0					
18.25838	15.53792	21.91166	36.47362	1					
0	0	0	0.96420	0					
0.32520	0.59840	0.68590	0.37990	0					
0	0	0	0	1					
					1	1	1	1	0
					0	0	0	0	1
79.34838	-78.26792	-69.66166	-128.27362	0					
					24.62984	26.13337	25.16961	38.84855	19.
					0.38970	0.51480	0.76820	1.33860	0
					0	0	0	0	0.
					0	0	0	0	0
					-66.50984	-73.71337	-58.39961	-92.94855	-79.
0	0	0	0	0	0	0	0	0	0
61.09	-62.73	-47.75	-91.80	-M	-41.88	-47.58	-33.23	-54.10	-60.

	Year 4						
g e" es$_4$	Spring peanut$_4$ P_{30}	Jute$_4$ P_{31}	Fall sweet potatoes$_4$ P_{32}	2nd rice$_4$ P_{33}	Fall peanut$_4$ P_{34}	Ratoon sugar cane$_4$ P_{35}	Family living$_4$ P_{36}
	1	1	0	0	0	1	0
	0	0	1	1	1	1	0
7	25.16961	38.84855	19.35388	16.47019	23.22635	38.66203	1
0	0.76820	1.33860	0	0	0	0.96420	0
	0	0	0.32520	0.59840	0.68590	0.37990	0
	0	0	0	0	0	0	1
7	-58.39961	-92.94855	-79.46388	-76.78019	-69.65635	-129.06203	0
	0	0	0	0	0	0	0
	-33.23	-54.10	-60.11	-60.31	-46.43	-90.40	-M

Table 7. (Continued)

Year	Resource or activity		Resource or activity level P_0	Spring sweet potatoes$_5$ P_{37}	Spring "hu-tze" sweet potatoes$_5$ P_{38}	Spring peanut$_5$ P_{39}	Jute$_5$ P_{40}	Ye… si po
1	Spring land$_1$	P_{47}	200 ares					
1	Fall land$_1$	P_{48}	200 ares					
1	Operating capital$_1$	P_{49}	17,000 NT$					
1	Spring labor$_1$	P_{50}	190 days					
1	Fall labor$_1$	P_{51}	190 days					
1	Family living$_1$	P_{52}	10,486 NT$					
2	Spring land$_2$	P_{53}	200 ares					
2	Fall land$_2$	P_{54}	200 ares					
2	Operating capital$_2$	P_{55}	0					
2	Spring labor$_2$	P_{56}	195 days					
2	Fall labor$_2$	P_{57}	195 days					
2	Family living$_2$	P_{58}	10,706 NT$					
3	Spring land$_3$	P_{59}	200 ares					
3	Fall land$_3$	P_{60}	200 ares					
3	Operating capital$_3$	P_{61}	0					
3	Spring labor$_3$	P_{62}	215 days					
3	Fall labor$_3$	P_{63}	215 days					
3	Family living$_3$	P_{64}	12,270 NT$					
4	Spring land$_4$	P_{65}	200 ares					
4	Fall land$_4$	P_{66}	200 ares					
4	Operating capital$_4$	P_{67}	0					
4	Spring labor$_4$	P_{68}	220 days					
4	Fall labor$_4$	P_{69}	220 days					
4	Family living$_4$	P_{70}	12,750 NT$					
5	Spring land$_5$	P_{71}	200 ares	1	1	1	1	0
5	Fall land$_5$	P_{72}	200 ares	0	0	0	0	1
5	Operating capital$_5$	P_{73}	0	26.10763	27.70137	26.67978	41.17946	20.
5	Spring labor$_5$	P_{74}	225 days	0.38970	0.51480	0.76820	1.33860	0
5	Fall labor$_5$	P_{75}	225 days	0	0	0	0	0.
5	Family living$_5$	P_{76}	13,300 NT$	0	0	0	0	0
	Z_{jk}		0	0	0	0	0	0
	$Z_{jk} - C_{jk}$		0	-41.29	-46.89	-32.32	-51.84	-59.

	Year 5					Disposal activities
;	Fall sweet potatoes$_5$ P_{41}	2nd rice$_5$ P_{42}	Fall peanut$_5$ P_{43}	Ratoon sugar cane$_5$ P_{44}	Family living$_5$ P_{45}	Spring land$_1$ $P_{47} \cdots$
	0	0	0	1	0	
	1	1	1	1	0	
46	20.51511	17.45840	24.61993	40.98175	1	
50	0	0	0	0.96420	0	
	0.32520	0.59840	0.68590	0.37990	0	
	0	0	0	0	1	
	0	0	0	0	0	0
	-59.15	-57.98	-45.16	-89.02	-M	0

Table 8. Optimum 5-year plan for the case farm under Situation I

1	2	3		4	
Year	Available capital[a]	Disposal resources[b]		Optimum combination of	
	NT$				
1	17,000.00000	P_{47} Spring land	29.17157 ares	P_2 Spring "hu-tze" sweet potatoes	1
		P_{50} Spring labor	102.05974 days	P_6 2nd rice	2
		P_{51} Fall labor	70.32055 days		
2	28,576.42672	P_{55} Operating capital	9,022.52150 NT$	P_{11} Spring "hu-tze" sweet potatoes	
		P_{57} Fall labor	75.32000 days	P_{13} Jute	1
				P_{15} 2nd rice	2
3	32,803.17979	P_{61} Operating capital	10,863.35960 NT$	P_{20} Spring "hu-tze" sweet potatoes	
		P_{63} Fall labor	95.32000 days	P_{22} Jute	1
				P_{24} 2nd rice	2
4	32,982.97412	P_{67} Operating capital	9,905.77330 NT$	P_{29} Spring "hu-tze" sweet potatoes	
		P_{69} Fall labor	100.32000 days	P_{31} Jute	1
				P_{33} 2nd rice	2
5	32,831.50250	P_{73} Operating capital	7,891.55010 NT$	P_{38} Spring "hu-tze" sweet potatoes	
		P_{75} Fall labor	159.96000 days	P_{40} Jute	1
				P_{41} Fall sweet potatoes	2

[a]Available capital - operating capital available for crop production and family

[b]Disposal resources for any one year are the amount of resources not transferred

on of crops	5 Discounted net returns	6 Family living	7 Fixed expenditures	8 (5) - (6+7) Discounted net income	9 Limiting resources
ares	NT$	NT$	NT$	NT$	
' 170.82686 200.00000	22,062.38667	10,486.00000	10,590.00000	986.39000	P_{48} Fall land P_{49} Operating capital
," 88.27448 111.72552 200.00000	23,955.43441	10,706.00000	10,740.00000	2,509.43000	P_{53} Spring land P_{54} Fall land P_{56} Spring labor
," 63.99688 136.00312 200.00000	23,313.14549	12,270.00000	11,020.00000	23.15000	P_{59} Spring land P_{60} Fall land P_{62} Spring labor
" 57.92748 142.07252 200.00000	22,504.31283	12,750.00000	11,100.00000	-1,345.69000	P_{65} Spring land P_{66} Fall land P_{68} Spring labor
" 51.85808 148.14192 200.00000	21,541.30250	13,300.00000	11,160.00000	-2,918.70000	P_{71} Spring land P_{72} Fall land P_{74} Spring labor

ily living.

rred to the total supply of available resources in the following year.

limitation of operating capital that only 170.83 ares of spring "hu-tze" sweet potatoes are produced. The limiting resources for this plan are fall land and operating capital. NT$22,062.39 is the discounted net return in year 1. The discounted net income, after subtraction of family living costs NT$10,486 and fixed expenditures NT$10,590 from the discounted net return, is NT$986.39. The disposal resources in year 1 are 29.18 ares of spring land, 102.06 days of spring labor, 70.32 days of fall labor. As land and labor are flow resources, they are not transferred to the supplies of these resources for the following year.

<u>In year 2</u>, NT$28,576.43 of operating capital is available for crop production and family living. This is a function of the level of production in the previous year. It is also obtained by adding year 1 available operating capital for production plus discounted net returns minus disposal operating capital, if any. Family living costs in year 2 are NT$10,706, hence NT$17,870.43 is actually available for crop production. Optimum plan for year 2 is to produce 88.27 ares of spring "hu-tze" sweet potatoes, 111.73 ares of jute and 200 ares of second rice. The disposal resources are NT$9,022.52 of operating capital and 75.32 days of fall labor. The disposal or unused operating capital in any one year is not transferred to the supply of available operating capital for the following year. For the case farm, unused

operating capital in any one year is reserved for fixed expenditures. The limiting resources in this plan are spring land, fall land and spring labor. The net return for year 2 is NT$23,955.43. Discounted net income, after subtraction of family living costs NT$10,706 and fixed expenditures NT$10,740 from the discounted net return, is NT$2,509.43. This is greater than year 1, because all 200 ares of spring land is used for production.

In year 3, there is NT$32,803.18 of available operating capital, of which NT$20,053.18 is used for production, after subtraction of family living costs. All land is utilized to plant crops. Since spring labor is increased, more jute can be produced than in year 2. The optimum combination of crops for year 3 is 64.00 ares of spring "hu-tze" sweet potatoes, 136.00 ares of jute and 200 ares of second rice. The disposal resources are NT$9,905.77 of operating capital, 100.32 days of fall labor. The limiting resources in year 3 are the same as in year 2. The discounted net return for year 3 is NT$23,313.15. After subtraction of family living costs NT$12,270 and fixed expenditures NT$11,020 from the discounted net return, the discounted net income is NT$23.15.

In year 4, there is NT$20,232.97 of operating capital available for crop production after the deduction of family living costs from the available capital is made. The optimum combination of crops for year 4 is 57.93 ares of spring "hu-tze" sweet potatoes, 142.07 ares of jute and 200 ares of second rice. The disposal resources are NT$9,905.77 of operating capital and 100.32 days of fall labor. The limiting resources are still spring land, fall land and spring labor. As net return in year 4 is discounted back to year 1, the discounted net return is NT$22,504.31 which is less than it is in year 2 or 3. Also, family living costs and fixed expenditures are increased. The discounted net income is -NT$1,345.69.

<u>In year 5</u>, the last year of the plan, NT$32,831.50 of operating capital is available for production and family living. NT$19,531.50 is used for crop production. The discounted net revenue of fall sweet potatoes is higher than second rice in year 5, therefore, second rice is replaced by fall sweet potatoes. The production of spring crops in year 5 is the same as in year 4, but 6 ares of spring "hu-tze" sweet potatoes are substituted for jute, because of the increasing supply of spring labor. The disposal resources are NT$7,891.55 of operating capital and 159.96 days of fall labor. The discounted net return is decreased to NT$21,541.30 which is less than the previous four years. The amount of family living costs and fixed expenditure is the largest one in any of five years. The discounted net income is -NT$2,918.70. The limiting resources are again spring land, fall land, and spring labor.

Over the 5-year period, discounted net returns total NT$113,776.48, before family living costs and fixed expenditures are subtracted. Fall land is the principal limiting resource in the production of fall crops in each year. Spring land and spring labor are also the limiting resources except in year 1. Because the operating capital requirement of crops is low and production of crops is limited by spring land, fall land and spring labor, operating capital is not limitational in years 2 through 5. The most profitable crop is second rice for fall, and jute for spring. The disposal operating capital in each year is reserved for the fixed expenditures of the year.

b. Situation II: Optimum 5-year plan for the case farm with the hiring of spring labor The essential difference between Situations I and II is that in the latter it is possible to hire spring labor from years 1 to 5. Therefore, we added five more real activities as shown in Table 9 to Table 7 and form the original input-output matrix for a 5-year dynamic linear program under Situation II. Since the hiring of spring labor adds to the spring labor supply and operating capital is needed to pay for it, there is a negative coefficient in the spring labor row and a positive coefficient in the operating capital row, as presented in Table 9. Also, the net revenue from purchasing a resource will be negative. If the spring labor can be hired for NT$10 a day, the revenue

Table 9. Hiring spring labor activities for 5 years

Resource or activity		$c_{jk} \rightarrow$ Resource or activity level P_0	-10.00 Hiring spring $labor_1$ P_{77}	-9.63 Hiring spring $labor_2$ P_{78}	-9.26 Hiring spring $labor_3$ P_{79}	-8.88 Hiring spring $labor_4$ P_{80}	-8.50 Hiring spring $labor_5$ P_{81}
Spring land	P_{47}	200 ares	0				
Fall land	P_{48}	200 ares	0				
Operating capital	P_{49}	17,000 NT$	10				
Spring labor	P_{50}	190 days	-1				
Fall labor	P_{51}	190 days	0				
Family living	P_{52}	10,486 NT$	0				
Spring land	P_{53}	200 ares		0			
Fall land	P_{54}	200 ares		0			
Operating capital	P_{55}	0		10.5			
Spring labor	P_{56}	195 days		-1			
Fall labor	P_{57}	195 days		0			
Family living	P_{58}	10,706 NT$		0			
Spring land	P_{59}	200 ares			0		
Fall land	P_{60}	200 ares			0		
Operating capital	P_{61}	0			11.0		
Spring labor	P_{62}	215 days			-1		
Fall labor	P_{63}	215 days			0		
Family living	P_{64}	12,270 NT$			0		

Table 9. (Continued)

Resource or activity		$c_{jk} \rightarrow$ Resource or activity level P_0	-10.00 Hiring spring $labor_1$ P_{77}	-9.63 Hiring spring $labor_2$ P_{78}	-9.26 Hiring spring $labor_3$ P_{79}	-8.88 Hiring spring $labor_4$ P_{80}	-8.50 Hiring spring $labor_5$ P_{81}
Spring land	P_{65}	200 ares				0	
Fall land	P_{66}	200 ares				0	
Operating capital	P_{67}	0				11.5	
Spring labor	P_{68}	220 days				-1	
Fall labor	P_{69}	220 days				0	
Family living	P_{70}	12,750 NT$				0	
Spring land	P_{71}	200 ares					0
Fall land	P_{72}	200 ares					0
Operating capital	P_{73}	0					12.0
Spring labor	P_{74}	225 days					-1
Fall labor	P_{75}	225 days					0
Family living	P_{76}	13,300 NT$					0
z_{jk}		0	0	0	0	0	0
$z_{jk} - c_{jk}$		0	10.00	9.63	9.26	8.88	8.50

contributed by the act of hiring one day of the labor will be -NT$10 as shown in Table 9 in year 1. The wage of spring labor per day each year is projected and the net revenues are discounted in Table 9.

An optimum 5-year plan for the case farm under situation II is presented in Table 10.

<u>In year 1</u>, the optimum plan for year 1 is the same as under Situation I. Because of operating capital limitations, there is no spring labor hired.

<u>In year 2</u>, available operating capital NT$28,576.43 is transferred from year 1 for production and consumption. Because of increasing operating capital for production, all 200 ares of spring land are used. Since 72.72 days of spring labor are hired, the optimum combination of crops is 200 ares of jute and 200 ares of second rice. The disposal resources are NT$7,260.00 of operating capital and 75.32 days of fall labor. As under Situation I, the unused operating capital in any one year is not transferred to the following year; it is reserved for the fixed expenditures of that year. The limiting resources are spring and fall land. The discounted net return is NT$24,131.70. The discounted net income, after subtraction of family living costs and fixed expenditures from the net return, is NT$2,685.70.

<u>In year 3</u>, NT$34,678.70 of operating capital is available for crop production and family living. NT$22,408.70 is

the amount used for production. The optimum combination of crops is the same as in year 2, but only 52.72 days of spring labor are hired because of increasing supply of spring labor by family labor in year 3. The disposal resources are NT$11,391.28 of operating capital and 95.32 days of fall labor. The limitation of resources is the same as in year 2. The discounted net return is NT$23,347.81. Since the family living costs and fixed expenditures are increased, the discounted net income is only NT$57.81.

In year 4, there is NT$34,273.50 of operating capital available for crop production. Since the shadow price (17, p. 85) of spring labor in year 4 is NT$7.91 per day, and the cost of hiring spring labor is NT$8.88 per day, there is no spring labor hired. The optimum plan for the year is the same as under Situation I, except that the disposal operating capital is NT$11,196.30. The limiting resources are spring and fall land.

In year 5, since the shadow price of spring labor in year 5 is NT$6.01 per day, and the cost of hiring spring labor is NT$8.50 per day, there is no spring labor hired. The available operating capital for production, optimum combination of crops and disposal resources are exactly the same as under Situation I. The discounted net returns is NT$21,941.30. The discounted net income, after subtraction of family living costs and fixed expenditures from the discounted net return, is -NT$2,518.70.

Over the 5-year period, the discounted net returns total, NT$113,987.51, is only NT$211.03 more than under Situation I, which does not hire spring labor. We can see that the activity of hiring spring labor does not increase net returns substantially. Due to the fact that we consider family living costs and fixed expenditures as fixed costs which are increasing year by year, the discounted net incomes are negative in years 4 and 5. Both spring and fall land, except spring land in year 1, are limiting resources. Operating capital is a limiting resource only for year 1. The most profitable crops are second rice in fall, and jute and spring "hu-tze" sweet potatoes in spring.

c. Situation III: Optimum 5-year plan for the case farm with the renting of land (not more than one hectare from years 3 through 5) This situation differs from Situation I in that land is rented. Since land is the most limiting resource under Situations I and II, the operator plans to rent additional land in order to enlarge his farm size and to maximize profits for the farm. Because land is the most scarce resource in the region, it will be difficult for him to rent more than one hectare of land. He also considers the supplies of spring labor and operating capital, and he plans to rent one hectare of land for years 3 through 5. This means that he begins to rent one hectare of the land at the beginning of year 3 and end it at the end of year 5. For the purpose of simplifying computations, it is assumed that the rent of years 3 to 5 is paid in year 3. In so doing we add both real and disposal activities of renting land in year 3, and the row restricting the renting of land to one hectare is shown in Table 11, which is added to Table 7 and forms the original input-output matrix for a 5-year dynamic linear program under Situation III. Renting land activity adds to the supplies of spring and fall land and operating capital is needed to pay the rent, therefore, negative coefficients are in the rows of spring and fall land, and a positive coefficient in the operating capital row as indicated in Table 11. The amounts of rent per are from years 3 to 5 in Table 11 are projected on the basis of the price index in the previous 5 years and the operator's expectations. The net revenue from renting land is negative and is also discounted.

An optimum 5-year plan for the case farm under Situation III is presented in Table 12.

<u>In years 1 and 2</u>, since there is no land renting activity introduced in years 1 and 2, the optimum plan for the years is the same as under Situation I.

<u>In year 3</u>, the available operating capital NT$32,803.18 is transferred from year 2 for production and family living. With the additional one hectare of land, the optimum combination of crops is 226.49 ares of spring "hu-tze" sweet

Table 11. Renting land activity for years 3 through 5

Resource or activity		$c_{jk} \rightarrow$ Resource or activity level P_0	Disposal activity 0 Renting land P_{77}	Real activity -172.83 Renting land P_{78}
Spring land	P_{47}	200 ares	0	0
Fall land	P_{48}	200 ares	0	0
Operating capital	P_{49}	17,000 NT$	0	0
Spring labor	P_{50}	190 days	0	0
Fall labor	P_{51}	190 days	0	0
Family living	P_{52}	10,486 NT$	0	0
Spring land	P_{53}	200 ares	0	0
Fall land	P_{54}	200 ares	0	0
Operating capital	P_{55}	0	0	0
Spring labor	P_{56}	195 days	0	0
Fall labor	P_{57}	195 days	0	0
Family living	P_{58}	10,706 NT$	0	0
Spring land	P_{59}	200 ares	0	-1
Fall land	P_{60}	200 ares	0	-1
Operating capital	P_{61}	0	0	64.53020
Spring labor	P_{62}	215 days	0	0
Fall labor	P_{63}	215 days	0	0
Family living	P_{64}	12,270 NT$	0	0
Spring land	P_{65}	200 ares	0	-1
Fall land	P_{66}	200 ares	0	-1
Operating capital	P_{67}	0	0	68.36730
Spring labor	P_{68}	220 days	0	0
Fall labor	P_{69}	220 days	0	0
Family living	P_{70}	12,750 NT$	0	0
Spring land	P_{71}	200 ares	0	-1
Fall land	P_{72}	200 ares	0	-1
Operating capital	P_{73}	0	0	72.44610
Spring labor	P_{74}	225 days	0	0
Fall labor	P_{75}	225 days	0	0
Family living	P_{76}	13,300 NT$	0	0
Renting land	P_{77}	100 ares	1	1
z_{jk}		0	0	0
$z_{jk} - c_{jk}$		0	0	172.83

Table 12. Optimum 5-year plan for the case farm under Situation III

1	2	3			4		
Year	Available capital	Disposal resources			Optimum combination of crops		
	NT$						Ares
1	17,000.00000	P_{47}	Spring land	29.17157 ares	P_2	Spring "hu-tze" sweet potatoes	170.82686
		P_{50}	Spring labor	102.05974 days	P_6	2nd rice	200.00000
		P_{51}	Fall labor	70.32055 days			
2	28,576.42672	P_{55}	Operating capital	9,022.52150 NT$	P_{11}	Spring "hu-tze" sweet potatoes	88.27448
		P_{57}	Fall labor	75.32000 days	P_{13}	Jute	111.72552
					P_{15}	2nd rice	200.00000
3	32,803.17980	P_{61}	Operating capital	1,140.74960 NT$	P_{20}	Spring "hu-tze" sweet potatoes	226.48788
		P_{63}	Fall labor	35.48000 days	P_{22}	Jute	73.51212
					P_{24}	2nd rice	300.00000
					P_{78}	Renting land	100.00000
4	46,843.01998	P_{67}	Operating capital	13,463.30330 NT$	P_{29}	Spring "hu-tze" sweet potatoes	220.41848
		P_{69}	Fall labor	40.48000 days	P_{31}	Jute	79.58152
					P_{33}	2nd rice	300.00000
5	46,678,83286	P_{73}	Operating capital	10,514.88010 NT$	P_{38}	Spring "hu-tze" sweet potatoes	214.34908
		P_{75}	Fall labor	127.44000 days	P_{40}	Jute	85.65092
					P_{41}	Fall sweet potatoes	300.00000

crops	5 Discounted net resources	6 Family living	7 Fixed expenditures	8 (5) - (6+7) Discounted net income	9 Limiting resources
es	NT$	NT$	NT$	NT$	
32686 00000	22,062.38667	10,486.00000	10,590.00000	986.38667	P_{48} Fall land P_{49} Operating capital
27448 72552 00000	23,955.43441	10,706.00000	10,740.00000	2,509.43441	P_{53} Spring land P_{54} Fall land P_{56} Spring labor
18788 51212 00000 00000	16,620.59402	12,270.00000	11,020.00000	-6,669.40598	P_{59} Spring land P_{60} Fall land P_{62} Spring labor
41848 58152 00000	32,885.87151	12,750.00000	11,100.00000	9,035.87151	P_{65} Spring land P_{66} Fall land P_{68} Spring labor
34908 55092 00000	32,235.97205	13,300.00000	11,160.00000	7,775.97205	P_{71} Spring land P_{72} Fall land P_{74} Spring labor

potatoes, 73.51 ares of jute and 300 ares of second rice. Since rent in years 3 through 5 is paid by operating capital for production and is subtracted from the net return, there is only NT$1,140.75 of disposal operating capital which is reserved for fixed expenditures of the year, and NT$16,620.59 of discounted net returns. The total amount of family living costs and fixed expenditures is greater than the discounted net returns, thus the discounted net income is -NT$8,669.41. Actually, the rent of years 4 and 5 is not paid in year 3, so that the amount of the rent should be added to the discounted net returns in year 3; the discounted net returns and the discounted net income would then be NT$28,472.59 and NT$4,182.59 respectively in year 3. The limiting resources are the same as in year 2.

<u>In year 4</u>, since one hectare of land is rented in year 3, there is one more hectare of land available to produce crops in spring and fall. Also, the available operating capital, which is transferred from year 3, is increased to NT$46,843.02 for production and consumption. NT$34,093.02 is the amount available for production. The optimum combination of crops is 220.42 ares of spring "hu-tze" sweet potatoes, 79.58 ares of jute and 300 ares of second rice. The disposal resources are NT$13,463.30 of operating capital and 40.48 days of fall labor. The discounted net return is NT$32,985.87. The discounted net income is NT$9,035.87.

If we subtract NT$5,754.00 of rent in year 4 from the amount of discounted net returns and discounted net income, the discounted net returns and the discounted net income would be NT$27,131.87 and NT$3,281.87 respectively. The limiting resources are still spring land, fall land and spring labor.

<u>In year 5</u>, after the deduction of family living costs from the available operating capital, there is NT$33,378.83 available for crop production in year 5. The optimum combination of crops is 214.35 ares of spring "hu-tze" sweet potatoes, 85.65 ares of jute and 300 ares of fall sweet potatoes. The disposal resources are NT$10,514.88 of operating capital and 127.44 days of fall labor. The discounted net returns is NT$32,235.97. The discounted net income, after subtraction of family living costs and fixed expenditures from the discounted net returns, is NT$7,775.97. If we subtract NT$6,098.00 of rent in year 5 from the amount of discounted net returns and discounted net income, the discounted net returns and the discounted net income would then be NT$26,137.97 and NT$1,677.97 respectively. The limiting of resources is the same as in year 4.

Over the 5-year period, the discounted net returns total NT$127,760.26 which exceeds the NT$13,983.78 under Situation I in which land is not rented. It is seen that the activity of renting land does increase substantially the amount of net return. It indicates that the renting of land is the most effective way to maximize profits for the farm. Since the rent of years 3 to 5 is charged to the discounted net returns in year 3, the discounted net income is -NT$6,669.41 in year 3. The limiting resources are the same as under Situation I. The most profitable crops are also the same, as under Situation I and II, but production is increased by one hectare of spring and fall crops.

d. Situation IV: Optimum 5-year plan for the case farm with the hiring of spring labor and the renting of land (not more than one hectare from years 3 to 5) This situation is the combination of Situations I, II and III. It is possible to hire spring labor from years 1 to 5 and to rent land, not more than one hectare from years 3 to 5, in addition to having the fixed resources. Tables 9 and 11 are combined as shown in Table 13 and added to Table 7 to form the original input-output matrix for a 5-year dynamic linear program for Situation IV.

An optimum 5-year plan for the case farm under Situation IV is presented in Table 14.

In year 1, since there is neither hiring of spring labor nor the renting of land, the optimum plan for the year is the same as under Situation I, II or III.

In year 2, since only the hiring of spring labor is introduced in year 2, the optimum plan is the same as under Situation II.

Table 13. Combination of renting land and hiring spring labor

Resource or activity		Resource or activity level P_0	Disposal activity $c_{jk} \rightarrow 0$ Renting land P_{77}	-172.83 Renting land P_{78}	-10.00 Hiring spring labor$_1$ P_{79}
Spring land	P_{47}	200 ares	0	0	0
Fall land	P_{48}	200 ares	0	0	0
Operating capital	P_{49}	17,000 NT$	0	0	10
Spring labor	P_{50}	190 days	0	0	-1
Fall labor	P_{51}	190 days	0	0	0
Family living	P_{52}	10,486 NT$	0	0	0
Spring land	P_{53}	200 ares	0	0	
Fall land	P_{54}	200 ares	0	0	
Operating capital	P_{55}	0	0	0	
Spring labor	P_{56}	195 days	0	0	
Fall labor	P_{57}	195 days	0	0	
Family living	P_{58}	10,706 NT$	0	0	
Spring land	P_{59}	200 ares	0	-1	
Fall land	P_{60}	200 ares	0	-1	
Operating capital	P_{61}	0	0	64.53020	
Spring labor	P_{62}	215 days	0	0	
Fall labor	P_{63}	215 days	0	0	
Family living	P_{64}	12,270 NT$	0	0	
Spring land	P_{65}	200 ares	0	-1	
Fall land	P_{66}	200 ares	0	-1	
Operating capital	P_{67}	0	0	68.36730	
Spring labor	P_{68}	220 days	0	0	
Fall labor	P_{69}	220 days	0	0	
Family living	P_{70}	12,750 NT$	0	0	
Spring land	P_{71}	200 ares	0	-1	
Fall land	P_{72}	200 ares	0	-1	
Operating capital	P_{73}	0	0	72.44610	
Spring labor	P_{74}	225 days	0	0	
Fall labor	P_{75}	225 days	0	0	
Family living	P_{76}	13,300 NT$	0	0	
Renting land	P_{77}	100 ares	1	1	
z_{jk}		0	0	0	0
$z_{jk} - c_{jk}$		0	0	172.83	10.00

Real activities				
-10.00	-9.63	-9.26	-8.88	-8.50
ing spring labor$_1$ P_{79}	Hiring spring labor$_2$ P_{80}	Hiring spring labor$_3$ P_{81}	Hiring spring labor$_4$ P_{82}	Hiring spring labor$_5$ P_{83}
0				
0				
10				
-1				
0				
0				
	0			
	0			
	10.5			
	-1			
	0			
	0			
		0		
		0		
		11.0		
		-1		
		0		
		0		
			0	
			0	
			11.5	
			-1	
			0	
			0	
				0
				0
				12.0
				-1
				0
				0
0	0	0	0	0
10.00	9.63	9.26	8.88	8.50

Table 14. Optimum 5-year plan for the case farm under Situation IV

1	2	3		4		5
Year	Available capital	Disposal resources		Optimum combination of crops		Discou net re
	NT$				Ares	N
1	17,000.00000	P_{47} Spring land	29.17157 ares	P_{2} Spring "hu-tze" sweet potatoes	170.82686	22,062
		P_{50} Spring labor	102.05974 days	P_{6} 2nd rice	200.00000	
		P_{51} Fall labor	70.32055 days			
2	28,576.42672	P_{55} Operating capital	7,259.99790 NT$	P_{13} Jute	200.00000	24,131
		P_{57} Fall labor	75.32000 days	P_{15} 2nd rice	200.00000	
				P_{78} Hiring spring labor	72.72052	
3	34,678.70000	P_{63} Fall labor	35.48000 days	P_{20} Spring "hu-tze" sweet potatoes	83.24808	16,698
				P_{22} Jute	216.75192	
				P_{24} 2nd rice	300.00000	
				P_{81} Hiring spring labor	117.99963	
				P_{78} Renting land	100.00000	
4	49,731.51501	P_{67} Operating capital	16,351.83740 NT$	P_{29} Spring "hu-tze" sweet potatoes	220.41848	32,885
		P_{69} Fall labor	40.48000 days	P_{31} Jute	79.58152	
				P_{33} 2nd rice	300.00000	
5	46,678.83286	P_{73} Operating capital	10,514.88010 NT$	P_{38} Spring "hu-tze" sweet potatoes	214.34908	32,235
		P_{75} Fall labor	127.44000 days	P_{40} Jute	85.65092	
				P_{41} Fall sweet potatoes	300.00000	

of crops	5 Discounted net returns	6 Family living	7 Fixed expenditures	8 (5) - (6+7) Discounted net income	9 Limiting resources
Ares	NT$	NT$	NT$	NT$	
	22,062.38667	10,486.00000	10,590.00000	986.38667	P_{48} Fall land
170.82686					P_{49} Operating capital
200.00000					
200.00000	24,131.70139	10,706.00000	10,740.00000	2,685.70139	P_{53} Spring land
200.00000					P_{54} Fall land
72.72052					
	16,698.18661	12,270.00000	11,020.00000	-6,591.81339	P_{59} Spring land
83.24808					P_{60} Fall land
216.75192					P_{61} Operating capital
300.00000					
17.99963					
00.00000					
	32,885.57151	12,750.00000	11,100.00000	9,035.57151	P_{65} Spring land
220.41848					P_{66} Fall land
79.58152					
300.00000					
	32,235.97205	13,300.00000	11,160.00000	7,775.97205	P_{71} Spring land
14.34908					P_{72} Fall land
85.65092					
00.00000					

<u>In year 3</u>, after the deduction of family living costs from the available operating capital NT$34,678.70 is made, there is NT$22,408.70 available for production; 118.00 days of hiring spring labor and 100.00 ares of renting land are introduced in the plan. The optimum combination of crops is 83.25 ares of spring "hu-tze" sweet potatoes, 216.75 ares of jute and 300.00 ares of second rice. From the discounted net returns is subtracted rent for the years 3 through 5; NT$16,698.19 of discounted net returns is left. The total amount of family living costs and fixed expenditures is greater than the discounted net returns. Thus, the discounted net income is -NT$6,591.81. But as the rent for the years 4 and 5 is actually not paid in year 3, it is permissible to add the amount of the rent to the discounted net returns of year 3. The discounted net returns and the discounted net income turn out to be NT$28,550.19 and NT$5,260.19 respectively. The limiting resources are spring land, fall land and operating capital.

<u>In year 4</u>, since 118 days of spring labor is hired and one hectare of land is rented to produce crops in year 3, the available operating capital, which is transferred from year 3, is increased to NT$49,731.52, of which NT$36,981.52 is available for production. Because the shadow price of spring labor in year 4 is NT$7.91 per day, and the cost of hiring spring labor is NT$8.88 per day, no spring labor is hired. As we

have one hectare of rented land from year 3, the optimum combination of crops and discounted net returns and discounted net income are the same as under Situation III. If NT$5,754.00 of rent is subtracted in year 4 from the amount of discounted net returns and discounted net income, the discounted net returns and the discounted net income are also the same as under situation III. The disposal resources are NT$16,551.84 of operating capital and 40.48 days of fall labor. The limiting resources are both spring and fall land.

In year 5, since the shadow price of spring labor in year 5 is NT$6.00 per day, and the cost of hiring spring labor is NT$8.88 per day, no spring labor is hired. One hectare of renting land from year 3 is utilized for crop production in the year. Because the available resources for the year are the same as in year 5 under Situation III, the optimum plan of the year is also the same as year 5 under Situation III.

Over the 5-year period, the discounted net returns total NT$128,013.82 which exceeds NT$14,237.34, NT$14,026.31 and NT$253.56 under Situations I, II and III respectively. Since the rent of years 3 to 5 is charged to the discounted net returns in year 3, the discounted net income is -NT$6,591.81 in year 3. The most profitable crops are second rice in fall and jute in spring. This is as it is under Situations I, II and III. Both spring and fall land and operating capital are the limiting resources.

B. The Separation Method

The above dynamic linear programming problem can be solved by the modified simplex method with an electronic computer. The largeness of some dynamic linear programs makes the magnitude of the matrices too cumbersome to solve the problem with a computer. In addition, the computer is not available in most countries. The separation method in this section and the decomposition algorithm and the functional equation approach in the following sections enable one to avoid these difficulties.

1. The method

The dynamic linear program can be decomposed into the form shown in Fig. 1. Fig. 1 shows that the originally large scale dynamic problem can be separated into individual subproblems which are tied together by joint constraints. The notation in Fig. 1 is explained as follows: A_t is the constraint matrix. The blocks B_t are the matrix of input-output coefficients. $b_1, \ldots, b_n$ is a column vector of resource restrictions and costs. The coefficients of the objective form are $c_1, c_2, \ldots, c_n$. Assume:

X_t, a variable n_t-vector, $t = 1, \ldots, n$

A_t, an m_t by n_t matrix

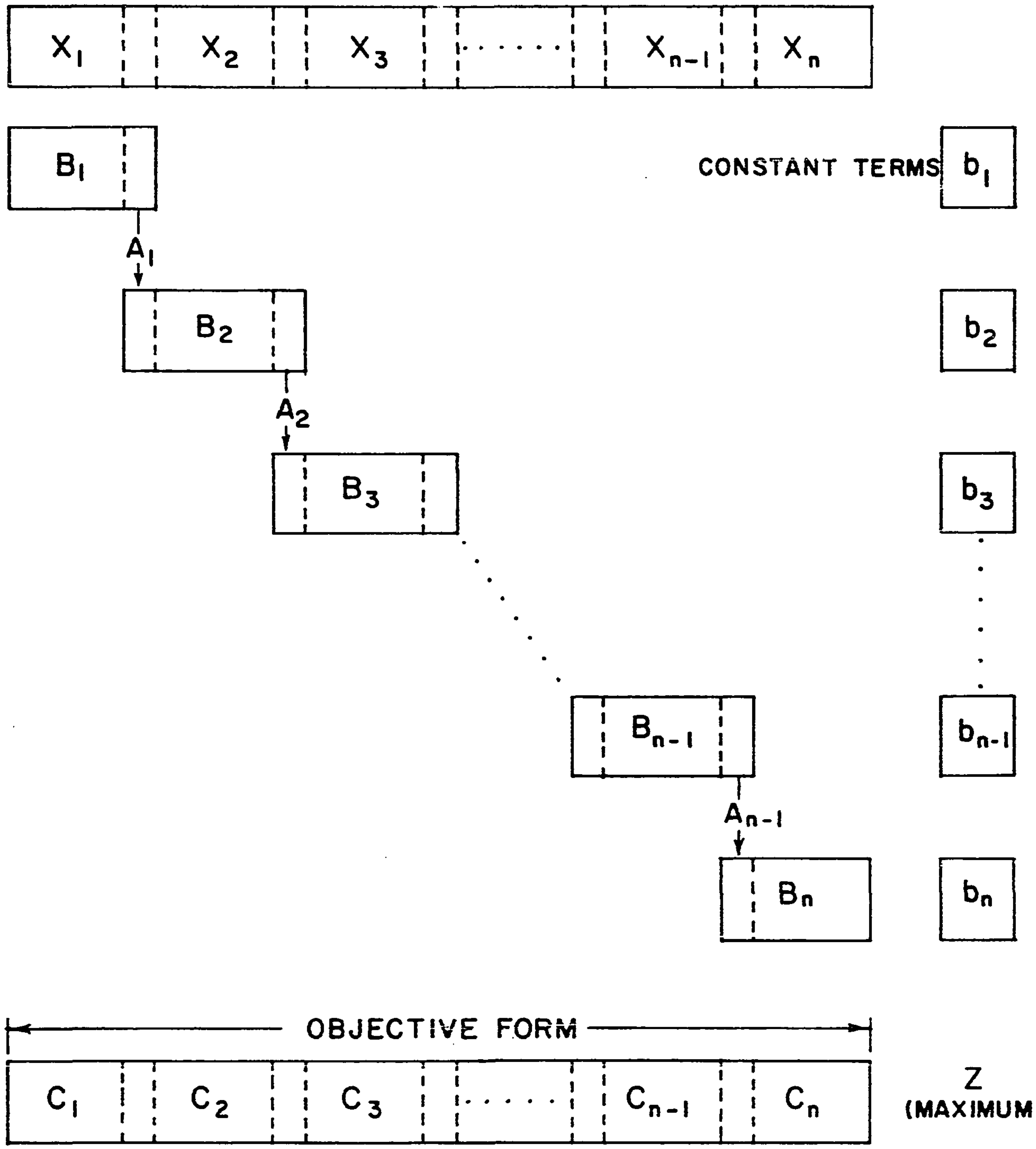

Fig. I Original dynamic problem

B_t, an m_t by n_t matrix,

c_t, an n_t-vector, the objective form

b_t, an n_t-vector, the right-hand side column of constants

A_1, ..., A_n are the inter-year activity coefficients; b_1 is the amount of resources available for production in year 1; b_2, ..., b_n are the resource restrictions which do not include inter-year transferable resources. The inter-year transferable resources available for year 2, ..., n depend upon the previous year's production and consumption. In order to solve the problem, we begin by finding the optimum program for the year 1 using the simplex method and obtain X_1, a vector of activity levels, and Z_1.

The production of year 2 depends on the amount of transferable resources of year 1. Since we know the inter-year activity coefficient A_1 and the activity level, X_1, in year 1, the amount of inter-year transferable resources is derivable: $A_1 \times X_1 = b_1'$. Next, b_1' and other restriction resources, b_2, are used as the supply of resources for year 2 production and X_2 and Z_2 are found by using the simplex method. Using the same procedure, one can find inter-year transferable resources for years 3, ..., n and their optimum activity levels of X_3, ..., X_n and Z_3, ..., Z_n.

Finally, the separate optimum plans are combined for years 1, 2, 3, ..., n and their value of Z_1, Z_2, Z_3, ..., Z_n, is added. This is the optimum plan for n years.

The method not only reduces a single N-dimensional problem to a sequence of N-one dimensional problems, but it also makes it possible to handle more variables (or joint constraint) with greater facility.

2. Numerical illustration

Let us take the matrix of the first three years in Table 7 as an instructive example. The inter-year activity coefficients of operating capital in years 1 and 2 are shared in common among the three time periods, i.e., selling products at the end of the first year will supply operating capital for production at the beginning of the second year; selling products at the end of the second year will supply operating capital for production at the beginning of the third year. They are joint constraints. There will be three subproblems in which each will be 6 x 16 in size. See Table 15.

Since operating capital is an inter-year transferable resource, the amount of operating capital available in years 2 and 3 depends upon the previous year's production and consumption.

As the amount of operating capital for year 1 and the inter-year activity coefficients of operating capital in years 1 and 2 are known, there will be no difficulty in finding the amount of operating capital available for production in years 2 and 3, and treating the subproblems 1, 2 and 3

Table 15. Three subproblems for the case farm programming

Resource or activity		Resource or activity level P_0	Spring $land_1$ P_{47}	Fall $land_1$ P_{48}	Operating $capital_1$ P_{49}	Spring $labor_1$ P_{50}	Fall $labor_1$ P_{51}	Family $living_1$ P_{52}	Sp sw pot. P
Spring $land_1$	P_{47}	200 ares	1						1
Fall $land_1$	P_{48}	200 ares		1					0
Operating $capital_1$	P_{49}	17,000 NT$			1				20.
Spring $labor_1$	P_{50}	190 days				1			0.
Fall $labor_1$	P_{51}	190 days					1		0
Family $living_1$	P_{52}	10,486 NT$						1	0
Z - C		0	0	0	0	0	0	0	-43.

Resource or activity		Resource or activity level P_0	Spring $land_2$ P_{53}	Fall $land_2$ P_{54}	Operating $capital_2$ P_{55}	Spring $labor_2$ P_{56}	Fall $labor_2$ P_{57}	Family $living_2$ P_{58}	Spr swe pota P
Spring $land_2$	P_{53}	200 ares	1						1
Fall $land_2$	P_{54}	200 ares		1					0
Operating $capital_2$	P_{55}	0			1				21.
Spring $labor_2$	P_{56}	195 days				1			0.
Fall $labor_2$	P_{57}	195 days					1		0
Family $living_2$	P_{58}	10,706 NT$						1	0
Z - C		0	0	0	0	0	0	0	-43.

Resource or activity		Resource or activity level P_0	Spring $land_3$ P_{59}	Fall $land_3$ P_{60}	Operating $capital_3$ P_{61}	Spring $labor_3$ P_{62}	Fall $labor_3$ P_{63}	Family $living_3$ P_{64}	Spr swe pota P
Spring $land_3$	P_{59}	200 ares	1						1
Fall $land_3$	P_{60}	200 ares		1					0
Operating $capital_3$	P_{61}	0			1				23.
Spring $labor_3$	P_{62}	215 days				1			0.
Fall $labor_3$	P_{63}	215 days					1		0
Family $living_3$	P_{64}	12,270 NT$						1	0
Z - C		0	0	0	0	0	0	0	-42.4

Subproblem 1

1	Spring sweet potatoes$_1$ P_1	Spring "hu-tze" sweet potatoes$_1$ P_2	Spring peanut$_1$ P_3	Jute$_1$ P_4	Fall sweet potatoes$_1$ P_5	2nd rice$_1$ P_6	Fall peanut$_1$ P_7	Ratoon sugar cane$_1$ P_8	Family living$_1$ P_9
	1	1	1	1	0	0	0	0	0
	0	0	0	0	1	1	1	1	0
	20.67970	21.94210	21.13290	32.61800	16.24990	13.82870	19.50130	32.46140	1
	0.38970	0.51480	0.76820	1.33860	0	0	0	0.96420	0
	0	0	0	0	0.32520	0.59840	0.68590	0.37990	0
	0	0	0	0	0	0	0	0	1
	-43.70	-49.69	-36.13	-61.46	-63.09	-67.87	-50.49	-94.66	-M

Subproblem 2

2	Spring sweet potatoes$_2$ P_{10}	Spring "hu-tze" sweet potatoes$_2$ P_{11}	Spring peanut$_2$ P_{12}	Jute$_2$ P_{13}	Fall sweet potatoes$_2$ P_{14}	2nd rice$_2$ P_{15}	Fall peanut$_2$ P_{16}	Ratoon sugar cane$_2$ P_{17}	Family living$_2$ P_{18}
	1	1	1	1	0	0	0	1	0
	0	0	0	0	1	1	1	1	0
	21.92048	23.25862	22.40087	34.57508	17.22489	14.65842	20.67138	34.40908	1
	0.38970	0.51480	0.76820	1.33860	0	0	0	0.96420	0
	0	0	0	0	0.32520	0.59840	0.68590	0.37990	0
	0	0	0	0	0	0	0	0	1
	-43.09	-48.98	-35.14	-58.91	-62.09	-65.25	-49.10	-93.22	-M

Subproblem 3

3	Spring sweet potatoes$_3$ P_{19}	Spring "hu-tze" sweet potatoes$_3$ P_{20}	Spring peanut$_3$ P_{21}	Jute$_3$ P_{22}	Fall sweet potatoes$_3$ P_{23}	2nd rice$_3$ P_{24}	Fall peanut$_3$ P_{25}	Ratoon sugar cane$_3$ P_{26}	Family living$_3$ P_{27}
	1	1	1	1	0	0	0	1	0
	0	0	0	0	1	1	1	1	0
	23.23570	24.65413	23.74492	36.64958	18.25338	15.53792	21.91166	36.47362	1
	0.38970	0.51480	0.76820	1.33860	0	0	0	0.96420	0
	0	0	0	0	0.32520	0.59840	0.68590	0.37990	0
	0	0	0	0	0	0	0	0	1
	-42.48	-48.28	-34.17	-56.45	-61.09	-62.73	-47.75	-91.80	-M

separately.

The step-by-step procedure for the separation method is as follows:

(a) The optimum plan of subproblem 1 in Table 15 is found by using the simplex method. The result is as follows:

P_2 = 170.83, P_6 = 200.00, P_9 = 10,486.00,

P_{47} = 29.17, P_{50} = 102.06, P_{51} = 70.32,

Z - C = 22,062.39

(b) To find the amount of operating capital supplied by year 1 for the production of year 2. The inter-year activity coefficients of operating capital in year 1 for P_2 and P_6 are -NT$71.63210 and -NT$81.69870 respectively, as seen in Table 7. This indicates that the amount contributed to the operating capital supply of year 2 by each are of P_2 and P_6 is NT$71.63210 and NT$81.69870 respectively. The optimum production of year 1 is 170.83 ares of P_2 and 200.00 ares of P_6. Therefore, the amount of operating capital available for year 2 is as follows:

(NT$71.63210 x 170.83) + (NT$81.69870 x 200.00) =

NT$28,576.91

(c) To place NT$28,576.91 in the operating capital supply row under the P_0 column of subproblem 2 in Table 15. Application of the simplex method finds optimum solution of subproblem 2.

P_{11} = 88.27, P_{13} = 111.73, P_{15} = 200.00,

P_{18} = 10,706.00, P_{55} = 9,022.52, P_{57} = 75.32,

Z - C = 23,955.43

There is a limited amount of spring land and spring labor, and as a result, there is NT$9,022.52 of operating capital unused.

(d) To find the amount of operating capital supplied by year 2 for the production of year 3. Since the inter-year activity coefficients of operating capital in year 2 for P_{11}, P_{13} and P_{15} are -72.23862, -93.48508 and -79.90842 respectively, as seen in Table 7. This indicates that the amount contributed to the capital supply of year 3 by each are of P_{11}, P_{13} and P_{15} is NT$72.23862, NT$93,48508 and NT$79.90842 respectively. The optimum production of year 2 is 88.27 ares of P_{11}, 111.73 ares of P_{13} and 200.00 ares of P_{15}. The amount of operating capital available for year 3 is as follows:

(NT$72.23862 x 88.27) + (NT$93.48508 x 111.73) +
(NT$79.90842 x 200.00) = NT$32,803.27

(e) To place NT$32,803.27 in the operating capital supply row under the P_0 column of subproblem 3 in Table 15. Application of the simplex method finds optimum solution of subproblem 3.

$P_{20} = 64.00$, $P_{22} = 136.00$, $P_{24} = 200.00$, $P_{27} = 12,270.00$

$P_{61} = 10,863.36$, $P_{63} = 95.32$,

$Z - C = 23,313.15$

It is due to the limited supply of resources, except fall labor, that there is NT$10,863.36 of operating capital unused. No operating capital transfer activity is required in the activities of year 3 because this is the final year of the plan.

(f) The optimum 3-year farm plan for the case farm is just to combine the separate optimum farm plans in years 1, 2 and 3 and add their value of Z - C. The optimum 3-year farm plan is shown as follows:

Year 1

$P_2 = 170.83$, $P_6 = 200.00$, $P_9 = 10,486.00$,

$P_{47} = 29.17$, $P_{50} = 102.06$, $P_{51} = 70.32$,

$Z - C = 22,062.39$

Year 2

$P_{11} = 88.27$, $P_{13} = 111.73$, $P_{15} = 200.00$,

$P_{18} = 10,706.00$, $P_{55} = 9,022.52$, $P_{57} = 75.32$

$Z - C = 23,955.43$

Year 3

$P_{20} = 64.00$, $P_{22} = 136.00$, $P_{24} = 200.00$,

$P_{27} = 12,270.00$, $P_{61} = 10,863.36$, $P_{63} = 95.32$

$Z - C = 23,313.15$

Total Z - C = 22,062.39 + 23,955.43 + 23,313.15

= 69,330.97

C. The Decomposition Algorithm

1. The decomposition principle

Dantzig's decomposition algorithm is one of the most useful approaches to solve the large scale dynamic program. There are many farm programming problems for a series of years.

> . . . they may be described, in part, as composed of separate linear programming problems tied together by a number of constraints considerably smaller than the total number imposed on the problem. (12, p. 767)

This structural property makes possible the decomposition of the problem into a sequence of small linear programs, through the joint constraints to coordinate the subprograms. The dynamic linear programming problem in a decomposed form is illustrated in Fig. 2.

In Fig. 2, the matrix of coefficients of the program is displayed. The constraint matrix is partitioned into non-zero blocks A_j and B_j. b, b_1, ..., b_n are the right-hand side column of constraints. c_1, c_2, ..., c_n are the costs. x_1, ..., x_n are activities. The original problem is posed as a linear programming problem. The task is to find the vector x_t ($t = 1, \ldots, n$) in order to satisfy the constraints:

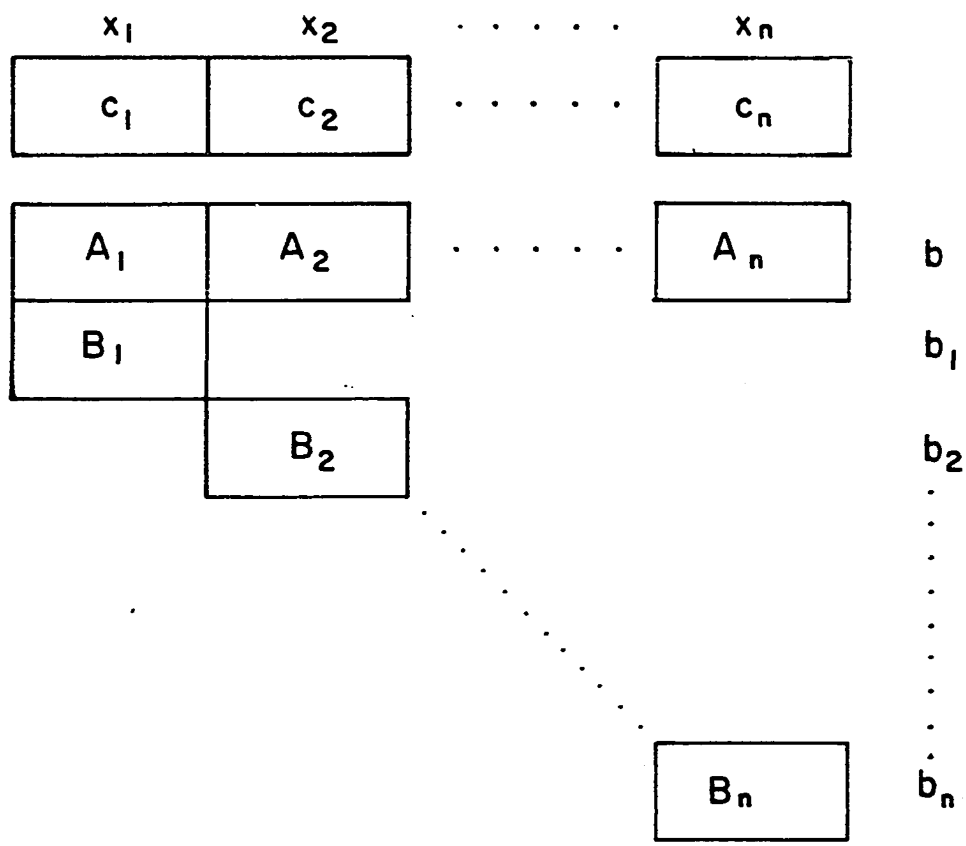

Fig. 2 The original problem in a decomposed form

(1) $\sum_t A_t x_t = b$ and $x_t \geq 0$ (all t) and

(2) $$B_t x_t = b_t \text{ (all t)}$$

which minimize the linear form:

(3) $$\sum_t c_t x_t$$

where x_t = a column vector of n_t activities

A_t = an m by n_t matrix

B_t = an m_t by n_t matrix

b, b_t = the right-hand side column of constants;

b is an n-vector and b_t is an m_t-vector

c_t = an n_t row vector, the objective form

> It would seem that each of the n sets of constraints of (2) constitutes a 'subproblem' of secondary importance to the whole program, and that they should be studied mainly through the restrictions they impose on the activities of the 'joint' constraints (1). (12, pp. 767-768)

Therefore it is possible to form an equivalent extremal problem from the extreme points of the n sets defined by equation 2. This extremal problem is the master program which will coordinate the subproblems.

The extremal problem is presented as follows:

It is assumed that

(4) $$S_t = \{x_t \mid x_t \geq 0,\ B_t x_t = b_t\}$$

is bounded for each t with t = 1, ..., n. Let

$$W_t = \{x_{t1}, \ldots, x_{tR_t}\},$$

the set of all extreme points of the convex polyhedron S_t under the conditions:

$$x_t \geq 0, \; B_t \; x_t = b_t$$

Define:

(5) $$P_{tk} = A_t \; x_{tk}$$

$$c_{tk} = c_t \; x_{tk}$$

The extremal program is to find number s_{tk} ($t = 1, \ldots, n$; $k = 1, \ldots, R_t$) subject to:

(6) $\sum_{t,k} P_{tk} \; s_{tk} = b$ $\qquad$ $s_{tk} \geq 0$ (all t,k)

(7) $\sum_k s_{tk} = 1$ (all t)

which will minimize the linear form:

(8) $\sum_{t,k} c_{tk} \; s_{tk}$

> The relation of the extremal problem to the original problem lies in the fact that any point x_t of S_t, because it is bounded and a convex polyhedral set, may be written as a convex combination of its extreme points, that is, as $\sum_k x_{tk} \; s_{tk}$, where $\{s_{t1}, \ldots, s_{tk_t}\}$ satisfy (7); and the expressions (6) and (8) are just the expressions (1) and (3) of the decomposed problem rewritten in terms of the s_{tk}. (12, p. 769)

The above relation can be stated that if the numbers (s_{tk}) solve the extremal program, equations 6 to 8, then the vectors

(9) $$S_t = \sum_k x_{tk} \; s_{tk} \qquad t = 1, \ldots, n$$

solve the problem, equations 1 to 3.

The matrix of coefficients for the extremal problem is shown in Fig. 3.

The extremal problem has $m + n$ constraint equations.

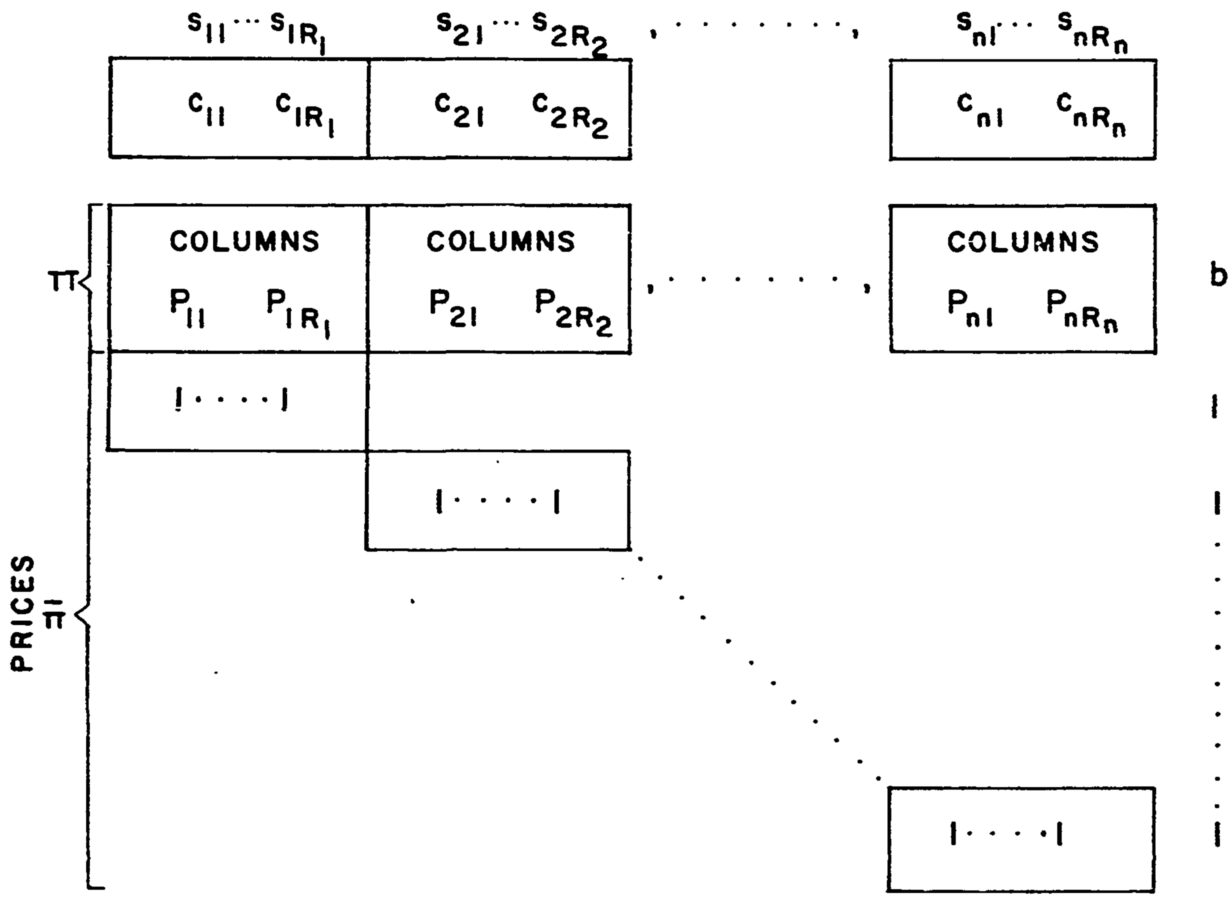

Fig. 3 The extremal problem

> The m joint constraints of the original problem have gone over into single constraints (6) constituting the upper block in Fig. [3], and the [m_t] constraints of the [t th] subproblem have gone over into single constraints of the form (7). The reduction in the total number of constraints is sizeable in case the [m_t] are large, and it is this fact on which the computational efficiency of the decomposition principle relies. The reduction appears to have been accomplished, however, by greatly enlarging the number of variables in the problem from the original [$\sum_t n_t$] to the number [$\sum_t R_t$]; the proposed method would be of little interest or value if it were not possible effectively to reduce this number. (12, p. 769)

It is the central idea of the decomposition algorithm that the extremal problem of equations 6 to 8 and Fig. 3 can be solved by the simplex method without prior calculation of the data of the problem. The procedure of a cycle of the decomposition algorithm will be presented as follows:

(a) Constitute a feasible basis for the extremal problem. Since the extremal problem has m + n equation constraints, a feasible basis will be constituted by m + n columns. The initial feasible basis is usually formulated by the m + n slack variables. There will be at hand the m + n-vector of prices (π; $\bar{\pi}$) - the m-vector π being associated with the first m constraints and the n-vector $\bar{\pi}$ with the remaining n, as shown in Fig. 3.

Since m + n slack variables have zero costs, the vector of initial prices associated with the initial feasible basis will be (π;$\bar{\pi}$) = (0, ..., 0;0).

Let $\bar{\pi}_t$ be the t-th component of $\bar{\pi}$ in the extremal problem, it can be written in the following manner:

(10) $$\pi P_{tk} + \overline{\pi}_t = c_{tk}$$

for the basic columns drawn from the t-th partition, where $t = 1, \ldots, n$.

(b) Use the prices associated with the feasible basis to form the modified costs for each subproblem as follows:

(11) $$C_t - \pi A_t$$

Minimize:

$$(C_t - \pi A_t)\ X_t$$

under the constraints:

$$X_t \geqslant 0$$

$$B_t\ X_t = b_t$$

(c) Find $\overline{X}_{t_o}$ which is the solution vector of the t-th subproblem, for which

(12) $$\delta_t = (C_{t_o} - \pi A_{t_o})\ \overline{X}_{t_o} - \overline{\pi}_{t_o} = \mathrm{Min}_t \left[(C_t - \pi A_t)\ \overline{X}_t - \overline{\pi}_t\right]$$

(d) If $\delta_t < 0$, the original problem is not solved, hence, form the new column and its associated cost for the extremal problem as

(13) $$(A_{t_o}\ \overline{X}_{t_o}\ ;\ 0, \ldots, 1, \ldots, 0) \text{ and } C_{t_o}\ \overline{X}_{t_o}$$

Add this new column and its associated cost to a new feasible basis, and delete one column from the basis in order to make the new basis feasible.

(e) Calculate the prices $(\pi ; \overline{\pi})$ from the above new basis, so that the next iterative step can be started.

(f) If $\delta \geq 0$ for all t, then s_{tk} solves the extremal problem; from equation 9, it is known that $S_t = \sum_k x_{tk} s_{tk}$, hence the original problem is also solved.

If the problem is for maximization, the δ_t in equation 12 will be changed to the following form:

$$(14) \qquad \delta_t = (\pi A_{t_o} - C_{t_o}) \overline{X}_{t_o} + \overline{\pi}_{t_o} = \text{Max}_t \left[(\pi A_t - C_t) \overline{X}_t - \overline{\pi}_t\right]$$

Also, the objective form of the subproblems will be $(\pi A_t - C_t) X_t$.

2. Application of the algorithm to the case farm

a. Case I: The capital demanded by subproblem 2 is smaller than the capital supplied by year 1 According to the decomposition algorithm, the programming matrix should first be ordered in the form shown in Fig. 2. The resultant matrix is given in Table 16. The first row of the table is the only row shared in common by the two time periods, i.e., selling products at the end of the first year will supply capital for production at the beginning of the second year. This single joint constraint and the two planning periods in the problem make up three rows for the extremal problem in the decomposition algorithm. Its basis will be a 3x3 matrix.

Table 16. ~~Simplex table for the case farm programming~~

Resource or activity		Resource or activity level P_0	Operating capital$_2$ P_{19}	Operating capital$_1$ P_{20}	Spring land$_1$ P_{21}	Fall land$_1$ P_{22}	Spring labor$_1$ P_{23}	Fall labor$_1$ P_{24}	Family living$_1$ P_{25}	Spr sw pot P
Operating capital$_2$	P_{19}	0	1							-64.
Operating capital$_1$	P_{20}	17,000 NT$		1						20.
Spring land$_1$	P_{21}	200 ares			1					1
Fall land$_1$	P_{22}	200 ares				1				0
Spring labor$_1$	P_{23}	190 days					1			0.
Fall labor$_1$	P_{24}	190 days						1		0
Family living$_1$	P_{25}	10,486 NT$							1	0
Spring land$_2$	P_{26}	200 ares								
Fall land$_2$	P_{27}	200 ares								
Spring labor$_2$	P_{28}	195 days								
Fall labor$_2$	P_{29}	195 days								
Family living$_2$	P_{30}	10,706 NT$								
Z - C		0	0	0	0	0	0	0	0	-43.

Resource or activity		Resource or activity level P_0	Spring sweet potatoes$_2$ P_{10}	Spring "hu-tze" sweet potatoes$_2$ P_{11}	Spring peanut$_2$ P_{12}	Jute$_2$ P_{13}	Fall sweet potatoes$_2$ P_{14}	2nd rice$_2$ P_{15}
Operating capital$_2$	P_{19}	0	21.92048	23.25862	22.40087	34.57508	17.22489	14.6584
Operating capital$_1$	P_{20}	17,000 NT$						
Spring land$_1$	P_{21}	200 ares						
Fall land$_1$	P_{22}	200 ares						
Spring labor$_1$	P_{23}	190 days						
Fall labor$_1$	P_{24}	190 days						
Family living$_1$	P_{25}	10,486 NT$						
Spring land$_2$	P_{26}	200 ares	1	1	1	1	0	0
Fall land$_2$	P_{27}	200 ares	0	0	0	0	1	1
Spring labor$_2$	P_{28}	195 days	0.38970	0.51480	0.76820	1.33860	0	0
Fall labor$_2$	P_{29}	195 days	0	0	0	0	0.32520	0.5984
Family living$_2$	P_{30}	10,706 NT$	0	0	0	0	0	0
Z - C		0	-43.09	-48.98	-35.14	-58.91	-62.09	-65.25

nily ving$_1$ P_{25}	Spring sweet potatoes$_1$ P_1	Spring "hu-tze" sweet potatoes$_1$ P_2	Spring peanut$_1$ P_3	Jute$_1$ P_4	Fall sweet potatoes$_1$ P_5	2nd rice$_1$ P_6	Fall peanut$_1$ P_7	Ratoon sugar cane$_1$ P_8	Family living$_1$ P_9
	-64.37970	-71.63210	-57.26290	-94.07800	-79.33990	-81.69870	-69.99130	-127.12140	0
	20.67970	21.94210	21.13290	32.61800	16.24990	13.82870	19.50130	32.46140	1
	1	1	1	1	0	0	0	1	0
	0	0	0	0	1	1	1	1	0
	0.38970	0.51480	0.76820	1.33860	0	0	0	0.96420	0
	0	0	0	0	0.32520	0.59840	0.68590	0.37990	0
1	0	0	0	0	0	0	0	0	1
0	-43.70	-49.69	-36.13	-61.46	-63.09	-67.87	-50.49	-94.66	-M

2	2nd rice$_2$ P_{15}	Fall peanut$_2$ P_{16}	Ratoon sugar cane$_2$ P_{17}	Family living$_2$ P_{18}	Spring land$_2$ P_{26}	Fall land$_2$ P_{27}	Spring labor$_2$ P_{28}	Fall labor$_2$ P_{29}	Family living$_2$ P_{30}
	14.65842	20.67138	34.40908	1					
	0	0	1	0	1				
	1	1	1	0		1			
	0	0	0.96420	0			1		
	0.59840	0.68590	0.37990	0				1	
	0	0	0	1					1
	-65.25	-49.10	-93.22	-M	0	0	0	0	0

Also, there will be two subproblems which will be 6x16 and 5x15 in size, as shown in Table 17.

Since this case farm study is a maximization problem, the criterion of δ_t in equation 14 should be used.

The step-by-step procedure for the decomposition algorithm is presented below:

(a) The initial feasible basis for the extremal problem is assumed to be the 3x3 identity matrix, hence, the price vector associated with the basis is: $(\pi;\bar{\pi}) = (0;0\ 0)$.

(b) Use $\pi = 0$ to form the modified objective functions $(\pi A_t - C_t)X_t$, $t = 1, 2$ for the two subproblems as shown in Table 17. Since $\pi = 0$ the original objective function of $-C_t\, X_t$ remains unchanged. Solving the subproblems by the simplex method, the following solutions are obtained:

The solution for subproblem 1:

$P_2 = 170.826$, $P_6 = 200.000$, $P_9 = 10,486$, $P_{21} = 29.174$,

$P_{23} = 102.590$, $P_{24} = 70.320$, $z - c = 22,062.38667$

The solution for subproblem 2:

$P_{11} = 88.275$, $P_{13} = 111.725$, $P_{15} = 200.000$,

$P_{18} = 10,706$, $P_{29} = 75.320$, $z - c = 23,955.43441$

(c) Test δ_t values by using equation 14:

$$\delta_{11} = (0 \cdot A_1 - C_1)\bar{X}_{11} + \bar{\pi} = -22,062.38667 + 0 < 0$$

where δ_{11} means δ value of subproblem 1 in the first run

$$A_1 = (-64.37970 \quad -71.63210 \quad -57.26290 \quad -94.07800 \quad -79.33990 \quad -81.69870 \quad -69.99130 \quad -127.12140 \quad 0)$$

Table 17. Two subproblems for the case farm in Case I

Resource or activity		Resource or activity level P_0	Operating capital P_{20}	Spring $land_1$ P_{21}	Fall $land_1$ P_{22}	Spring $labor_1$ P_{23}	Fall $labor_1$ P_{24}	Family $living_1$ P_{25}	Subpro Spring sweet $potatoes_1$ P_1	Spr "hu- swe pota P
Operating capital	P_{20}	17,000 NT$	1						20.67970	21.
Spring $land_1$	P_{21}	200 are		1					1	1
Fall $land_1$	P_{22}	200 are			1				0	0
Spring $labor_1$	P_{23}	190 days				1			0.38970	0.5
Fall $labor_1$	P_{24}	190 days					1		0	0
Family $living_1$	P_{25}	10,486 NT$						1	0	0
Z - C		0	0	0	0	0	0	0	-43.70	-49.6
Resource or activity		Resource or activity level P_0	Spring $land_2$ P_{26}	Fall $land_2$ P_{27}	Spring $labor_2$ P_{28}	Fall $labor_2$ P_{29}	Family $living_2$ P_{30}		Subprob Spring sweet $potatoes_2$ P_{10}	Spri "hu-t swee potat P_1
Spring $land_2$	P_{26}	200 are	1						1	1
Fall $land_2$	P_{27}	200 are		1					0	0
Spring $labor_2$	P_{28}	195 days			1				0.38970	0.51
Fall $labor_2$	P_{29}	195 days				1			0	0
Family $living_2$	P_{30}	10,706 NT$					1		0	0
Z - C		0	0	0	0	0	0		-43.09	-48.98

Subproblem 1

…ring …eet …atoes$_1$ …$_1$	Spring "hu-tze" sweet potatoes$_1$ P_2	Spring peanut$_1$ P_3	Jute$_1$ P_4	Fall sweet potatoes$_1$ P_5	2nd rice$_1$ P_6	Fall peanut$_1$ P_7	Ratoon sugar cane$_1$ P_8	Family living$_1$ P_9
.67970	21.94210	21.13290	32.61800	16.24990	13.82870	19.50130	32.46140	1
	1	1	1	0	0	0	1	0
	0	0	0	1	1	1	1	0
38970	0.51480	0.76820	1.33860	0	0	0	0.96420	0
	0	0	0	0.32520	0.59840	0.68590	0.37990	0
	0	0	0	0	0	0	0	1
70	-49.69	-36.13	-61.46	-63.09	-67.87	-50.49	-94.66	-M

Subproblem 2

…ing …et …toes$_2$ …10	Spring "hu-tze" sweet potatoes$_2$ P_{11}	Spring peanut$_2$ P_{12}	Jute$_2$ P_{13}	Fall sweet potatoes$_2$ P_{14}	2nd rice$_2$ P_{15}	Fall peanut$_2$ P_{16}	Ratoon sugar cane$_2$ P_{17}	Family living$_2$ P_{18}
	1	1	1	0	0	0	1	0
	0	0	0	1	1	1	1	0
…970	0.51480	0.76820	1.33860	0	0	0	0	0
	0	0	0	0.32520	0.59840	0.68590	0.37990	0
	0	0	0	0	0	0	0	1
	-48.98	-35.14	-50.91	-62.09	-65.25	-49.10	-93.22	-M

C_1 = (43.70 49.69 36.13 61.46 63.09
67.87 50.49 94.66 0)

$\overline{X}_{11}$ = (0 170.826 0 0 0
200 0 0 10,486)

where $\overline{X}_{11}$ is the solution of subproblem 1 in the first run, in terms of the real activities.

$$\delta_{21} = (0 \cdot A_2 - C_2)\overline{X}_{21} + \overline{\pi} = -23{,}955.43441 + 0 < 0$$

where δ_{21} means δ value of subproblem 2 in the first run

A_2 = (21.92048 23.25862 22.40087 34.57508
17.22489 14.65842 20.67138 34.40908 1)

C_2 = (43.09 48.98 35.14 58.91 62.09 65.25
49.10 93.22 0)

$\overline{X}_{21}$ = (0 88.275 0 111.725 0 200
0 0 10,706)

where $\overline{X}_{21}$ is the solution of subproblem 2 in the first run, in terms of the real activities.

Since both δ_1 and δ_2 are negative, it is necessary to form two new columns and their associated costs for the extremal problem, using equation 13

$$P_{11} = (A_1 \cdot \overline{X}_{11}; 1\ 0) = (-28{,}576.42672 \quad 1 \quad 0)$$

and

$$c_{11} = C_1 \cdot \overline{X}_{11} = 22{,}062.38667$$

$$P_{21} = (A_2 \cdot \overline{X}_{21}; 0\ 1) = (19{,}553.74538 \quad 0 \quad 1)$$

and

$$c_{22} = C_2 \cdot \overline{X}_{21} = 23{,}955.43441$$

(d) These two columns and their costs are added to the initial feasible basis of the extremal problem, and columns from the initial basis are deleted in accordance with the simplex rule. The solution of the problem as shown in Table 18 determines the new feasible basis as well as the level of the activities s_{tk} in the basis. The columns P_{11}, P_{21}, and I_3 now constitute a new basis, and the level of each activity will be $s_{11} = 1$, $s_{21} = 1.46$, and $I_3 = -0.46$ respectively.

(e) The price vector (see Table 18) associated with the new basis is now: $(\pi;\bar{\pi}) = (1.224;\ 57{,}037.32 \quad 0)$. This new price vector is used to indicate the second cycle of the decomposition algorithm.

(f) In beginning the second cycle, two new objective functions for the two subproblems are modified by using $\pi = 1.224$. They are:

$$(1.224A_1 - C_1)X_1 = (-122.501 \quad -137.368 \quad -106.220 \quad -160.202 \quad -167.869 \quad -136.159 \quad -250.257 \quad 0)\ X_1$$

$$(1.224A_2 - C_2)X_2 = (-16.259 \quad -20.511 \quad -7.721 \quad -16.590 \quad -41.007 \quad -47.308 \quad -51.103 \quad 1.224)\ X_2$$

The Z - C rows are next replaced in subproblems 1 and 2 by the two forms shown above. The remaximization of the two subproblems with new objective forms bring the following solutions:

Table 18. Determination of the current feasible basis for the extremal problem in Case I

	0	I_1	I_2	I_3	P_{11}	P_{21}
I_1	0	1	0	0	-28,576.42672	19,553.74538
I_2	1	0	1	0	1	0
I_3	1	0	0	1	0	1
Z-C	0	0	0	0	-22,062.38667	-23,955.43441
P_{21}	0	0.0000511	0	0	-1.46	1
I_2	1	0	1	1	1	0
I_3	1	-0.0000511	0	1	1.46	0
Z-C	0	1.22412	0	0	-57,037.32	0
P_{21}	1.46	0.0000511	1.46	0	0	1
P_{11}	1.00	0	1	0	1	0
I_3	-0.46	-0.0000511	-1.46	1	0	0
Z-C	57,037.32	1.22412	57,037.32	0	0	0

$\overline{X}_{12}$ = (0 170.826 0 0 0 200 0 0 10,486)

with P_{21} = 29.174; P_{23} = 102.58; P_{24} = 70.32, Z-C = 57,039.79

$\overline{X}_{22}$ = (0 88.275 0 111.725 0 200 0 0 10.706)

with P_{29} = 75.32, Z-C = 0

(g) Test δ_t values by using equation 14:

$$\delta_{12} = (1.224A_1 - C_1)X_1 + 57,037.32 = 0$$

$$\delta_{22} = (1.224A_2 - C_2)X_2 + 0 = 0$$

Now both δ_1 and δ_2 are equal to zero. The current solution (s_{11}; s_{21}) has solved the extremal problem. As the operating capital of NT$19,553.75 demanded by subproblem 2 is smaller than the supply of operating capital of NT$28,576.43 in subproblem 1, the surplus operating capital raises the level of activity s_{21} to 1.46 and causes the production of year 2 to be:

$$S_2' = s_{21}\,\overline{X}_{22} = 1.46\ (0 \quad 88.275 \quad 0 \quad 111.725 \quad 0 \quad 200 \quad 0 \quad 0 \quad 10,706)$$

$$= (0 \quad 128.882 \quad 0 \quad 163.107 \quad 0 \quad 292.000 \quad 0 \quad 0 \quad 15,630.760)$$

But it is an infeasible program, because the resources of land and labor are limited to produce the production as presented $\overline{X}_{22}$, hence s_{21} is forced equal to 1.

(h) Accordingly, the original problem is solved by equation 9, and the solution is as follows:

$$s_{11}\,\overline{X}_{12} = 1\ (0 \quad 170.826 \quad 0 \quad 0 \quad 0 \quad 200 \quad 0 \quad 0 \quad 10,486)$$

$s_{21}\ \bar{X}_{22} = 1$ (0 88.275 0 111.725 0 200
0 0 10,706)

These vectors indicate that the solution for the original problem is P_2 = 170.825; P_6 = 200.000; P_9 = 10,486; P_{11} = 88.275; P_{13} = 111.725; P_{15} = 200.000 and P_{18} = 10,706 in terms of real activities. The complete solution is shown below:

P_2 = 170.83; P_6 = 200.00; P_9 = 10,486.00; P_{11} = 88.28;
P_{13} = 111,725.00; P_{15} = 200.00; P_{18} = 10,706.00;
P_{19} = 9,022.68; P_{21} = 29.17; P_{23} = 102.06; P_{24} = 70.32;
P_{29} = 75.32; Z-C = 46,017.77

According to the above solution, the farmer should produce 170.83 ares of spring "hu-tze" sweet potatoes, 200.00 ares of second rice and spend NT$10,486.00 for family living in the first year. In the second year he should produce 88.28 ares of spring "hu-tze" sweet potatoes, 111.73 ares of jute, 200 ares of second rice and spend NT$10,706.00 for family living. He would have 29.17 ares of spring land, 102.06 days of spring labor and 70.32 days of fall labor unused in the first year and NT$9,022.68 of operating capital and 75.32 days of fall labor unused in the second year. The total net return for the two-year period would be NT$46,017.77 in terms of the present value.

From the above results, the conclusion is reached that if the supply of capital in year 1 is greater than the amount needed in year 2, then the optimum plans for year 1 and year 2 separately is the same as the optimum plan for the two-year period.

b. <u>Case II: The capital demanded by subproblem 2 is greater than the capital supply of year 1</u> We continue to use the above two years' example, while increasing the supply of resources in the second year as follows:

Spring land	450 ares
Fall land	450 ares
Spring labor	440 days
Fall labor	440 days

The step-by-step procedure for the decomposition algorithm is the same as in the Case I and is presented as follows:

(a) The initial feasible basis for the extremal problem is the same as step 1 of Case 1.

(b) Use $\pi = 0$ to form the modified objective functions $(\pi A_t - C_t)X_t$, $t = 1, 2$ for the two subproblems. In Table 17, the P_0 column for subproblem 2 changes as follows:

Spring land	P_{26}	450 ares
Fall land	P_{27}	450 ares
Spring labor	P_{28}	440 days
Fall labor	P_{29}	440 days
Family living	P_{30}	10,706 NT$

Since $\pi = 0$, the original objective function of $-C_t X_t$

remains unchanged. Solving the subproblems by the simplex method, the following solutions are obtained:

The solution for subproblem 1:

$P_2 = 170.825$, $P_6 = 200.000$, $P_9 = 10{,}486.000$,

$P_{21} = 29{,}174.000$, $P_{23} = 102.590$, $P_{24} = 70.320$,

$Z-C = 22{,}062.29$

The solution for subproblem 2:

$P_{11} = 197.099$, $P_{13} = 252.901$, $P_{15} = 450.000$,

$P_{18} = 10{,}706$, $P_{29} = 170.720$, $Z-C = 53{,}914.80$

(c) Test δ_t values by using equation 14

$$\delta_{11} = (0 \cdot A_1 - C_1)\overline{X}_{11} + \overline{\pi} = -22{,}062.29 + 0 < 0$$

where $A_1 = (-64.37970 \quad -71.63210 \quad -57.26290 \quad -94.07800 \quad -79.33290 \quad -81.69870 \quad -69.99130 \quad -127.12140 \quad 0)$

$C_1 = (43.70 \quad 49.69 \quad 36.13 \quad 61.46 \quad 63.09 \quad 67.87 \quad 50.49 \quad 94.66 \quad 0)$

$\overline{X}_{11} = (0 \quad 170.825 \quad 0 \quad 0 \quad 0 \quad 200 \quad 0 \quad 0 \quad 10{,}486)$

$$\delta_{21} = (0 \cdot A_1 - C_1)\overline{X}_{21} + \overline{\pi} = -53{,}914.80$$

where $A_2 = (21.92048 \quad 23.25862 \quad 22.40087 \quad 34.57508 \quad 17.22489 \quad 14.65842 \quad 20.67138 \quad 34.40908 \quad 1)$

$C_2 = (43.09 \quad 48.98 \quad 35.14 \quad 58.91 \quad 62.09 \quad 65.15 \quad 49.10 \quad 93.22 \quad 0)$

$\overline{X}_{21} = (0 \quad 197.099 \quad 0 \quad 252.901 \quad 0 \quad 450 \quad 0 \quad 0 \quad 10{,}706)$

Since both δ_{11} and δ_{21} are negative, it is necessary to form two new columns and their associated costs for the extremal problem, using equation 13:

$$P_{11} = (A_1\ \bar{X}_{11};\ 1 \quad 0) = (-28{,}576.29348 \quad 1 \quad 0)$$

and

$$c_{11} = C_1 \cdot \bar{X}_{21} = 22{,}062.294$$

$$P_{21} = (A_2 \cdot \bar{X}_{21};\ 0 \quad 1) = (30{,}630.68120 \quad 0 \quad 1)$$

and

$$c_{21} = C_2 \cdot \bar{X}_{21} = 53{,}914.82679$$

(d) These two columns and their costs are added to the initial feasible basis of the extremal problem. The solution of the problem as shown in Table 19 determines the new feasible basis as well as the level of the activities s_{tk} in the basis. The columns P_{11}, P_{21} and I_3 now constitute a new basis, and the level of each activity will be $s_{11} = 1$, $s_{21} = 0.933$ and $I_3 = 0.067$ respectively.

(e) The price vector (see Table 19) associated with the new basis is now: $(\pi ; \bar{\pi}) = 1.76;\ 72{,}364.80 \quad 0)$. This new price vector is used to indicate the second cycle of the decomposition algorithm.

(f) In beginning the second cycle, two new objective functions for the two subproblems are modified by using $\pi = 1.76$. They are:

$$(1.76A_1 - C_1)X_1 = (-157.008 \quad -175.762 \quad -136.913$$
$$-227.037 \quad -202.728 \quad -211.660 \quad -173.675$$
$$-318.394 \quad 0)\ X_1$$

Table 19. Determination of the current feasible basis for the extremal problem in Case II

	0	I_1	I_2	I_3	P_{11}	P_{21}
I_1	0	1	0	0	-28,576.29	30,630.68
I_2	1	0	1	0	1	0
I_3	1	0	0	1	0	1
Z-C		0	0	0	-22,062.29	-53,914.80
P_{21}	0	0.0000326	0	0	-0.933	1
I_2	1	0	1	0	1	0
I_3	1	-0.0000326	0	1	0.933	0
Z-C	0	1.75762	0	0	-72,364.80	0
P_{21}	0.933	0.0000326	0.933	0	0	1
P_{11}	1.000	0	1	0	1	0
I_3	0.067	-0.0000326	-0.933	1	0	0
Z-C	72,364.80	1.75762	72,364.80	0	0	0

$$(1.76A_2 - C_2)X_2 = (-4.51 \quad -8.045 \quad 4.286 \quad 1.942$$
$$-31.774 \quad -39.451 \quad -12.718 \quad -32.660 \quad 1.76)\ X_2$$

Before the remaximization of the two subproblems with new objective forms, we use $\overline{X}_{11}$ and $\overline{X}_{21}$ to test δ_t values by using equation 14 in order to determine whether we need further remaximization of the two subproblems or not. The δ_t values are tested as follows:

$$\delta_{12} = (1.76A_1 - C_1)\overline{X}_{11} + 72{,}364.80 = 0$$
$$\delta_{22} = (1.76A_2 - C_2)\overline{X}_{21} + 0 = 0$$

Both δ_{11} and δ_{22} are equal to zero; the current solution $(s_{11};\ s_{21})$ has solved the extremal problem.

Accordingly, the original problem is solved as follows:

$$s_{11}\ \overline{X}_{11} = 1\ (\ 0 \quad 170.825 \quad 0 \quad 0 \quad 0 \quad 200 \quad 0$$
$$0 \quad 10{,}486)$$

$$s_{21}\ \overline{X}_{21} = 0.933\ (\ 0 \quad 197.099 \quad 0 \quad 252.901 \quad 0$$
$$450 \quad 0 \quad 0 \quad 10{,}706)$$
$$= (\ 0 \quad 183.893 \quad 0 \quad 235.957 \quad 0 \quad 419.850$$
$$0 \quad 0 \quad 9{,}988.698)$$

These vectors indicate that the solution for the original problem is $P_2 = 170.825$, $P_6 = 200$, $P_9 = 10{,}486$, $P_{11} = 183.893$, $P_{13} = 235.957$, $P_{15} = 419.850$ and $P_{18} = 9{,}988.698$ in terms of real activity. The results of the complete solution is as follows:

$P_2 = 170.825$, $P_6 = 200.000$, $P_9 = 10{,}486.000$,

$P_{23} = 102.059$, $P_{24} = 70.320$, $P_{21} = 29.175$,

$P_{11} = 183.893$, $P_{13} = 235.957$, $P_{15} = 419.85$,

$P_{18} = 9{,}988.698$, $P_{26} = 30.150$, $P_{27} = 30.150$,

$P_{28} = 29.480$, $P_{29} = 188.760$, $Z-C = 72{,}364.81$

From the above solution the case farm should produce 170.825 ares of spring "hu-tze" sweet potatoes, 200 ares of second rice and spend NT$10,486 for family living in the first year. In the second year the farm should produce 183.893 ares of spring "hu-tze" sweet potatoes, 235.957 ares of jute and 419.85 ares of second rice and spend NT$9,988.698 for family living. He would have 29.175 ares of spring land, 102.059 days of spring labor and 70.32 days of fall labor unused in the first year and 30.150 ares of spring land, 30.150 ares of fall land, 29.48 days of spring labor and 188.76 days of fall labor unused in the second year. The total net return for the two-year period would be NT$72,364.81 in terms of the present value. The application of this algorithm was also illustrated by Chou (7).

D. The Functional Equation Approach

1. Principle of optimality

The functional equation approach of dynamic programming was developed by Bellman. It was proved useful when applied to multi-stage decision process problems. The fundamental concept of this approach is Bellman's "Principle of optimality". The principle states that

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. (2, p. 83)

This principle can also be written in mathematical form as follows (13, 14):

$$f_N\ (X) = \underset{P}{\mathrm{Max}} \left[R_N\ (P,\ X) + f_{N-1}\ (X'\ (P)) \right] \tag{1}$$

where f_N (X) is the optimum return from an N-stage process starting with an initial quantity X.

X is a state vector indicating the resources available within the first stage and the subsequent stages.

R_N (P, X) is the yield during the first stage of the N stage process and depends on decision P and initial state X.

X' is the transformed state vector resulting from the choice of policy P.

f_{N-1} (X' (P)) is the optimal return from (N-1) stage starting in a state X' which depends on the old state X and policy P.

Once we have formulated the problem recursively, we may determine a function satisfying the relation or may iterate the equation numerically.

By this approach, we can represent the process in the form of a functional equation, and reduce a single N-dimensional problem to a sequence of N-one dimensional problems.

It makes the computation easier and reduces the time required to solve the original problem.

2. Slightly intertwined linear programming matrices

Bellman proposed the functional equation approach of dynamic programming to treat a linear programming problem involving a "slightly intertwined" matrix, i.e., one that is almost block diagonal (3), which was elaborated by Dantzig (10). According to this approach, an N-stage dynamic problem can be solved by a one-stage computation performed N times. It means that we have reduced a single N-dimensional problem to a sequence of N-one dimensional problems. The time required to solve the original problem is greatly reduced through the reduction of dimension.

Dantzig used a four-stage dynamic system as an example. This system is shown in Fig. 4. There is only one variable s_i shared in common between the stages. In order to solve the problem, we begin to find the optimal program for the last stage by the variable resource linear programming technique (17, pp. 232-263). Using this technique, the s_3, which is treated as a parameter in Fig. 5, will start with a value of zero and can be varied over any specified range of values, yielding the return z_3 of the activities in the fourth stage to the objective form as a function of s_3.

z_3 as a function of s_3 is shown in Fig. 7. It is a

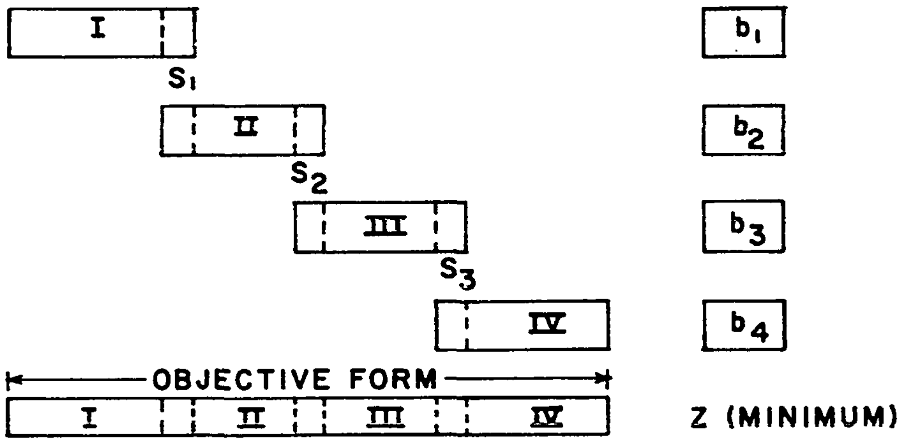

Fig. 4 A four stage dynamic system involving a slightly intertwined matrix

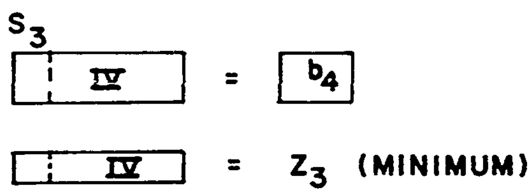

Fig. 5 Linear programming for the fourth stage

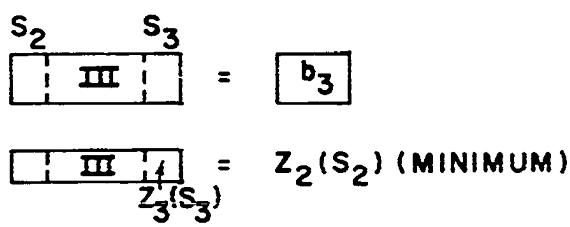

Fig. 6 The optimal program for the third stage

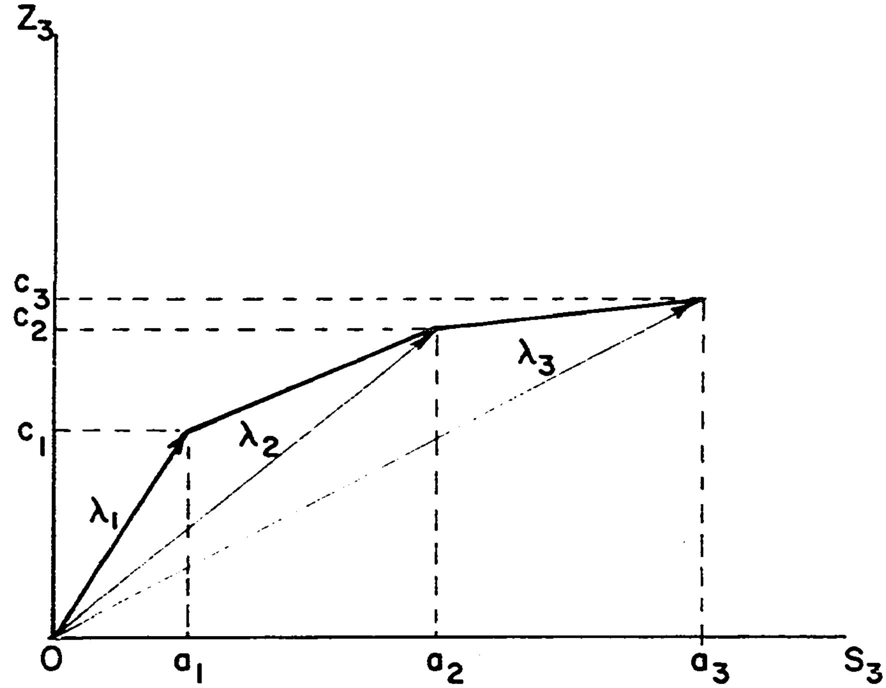

Fig. 7 Z_3 as a broken line convex function of S_3

broken line convex function of s_3 and only the values at the breakpoints are recorded; those values in-between are available by linear interpolation.

The s_3 and z_3 (s_3) can be written as follows:

(1) $s_3 = a_1 \lambda_1 + a_2 \lambda_2 + \cdots + a_{k+1} \lambda_{k+1}, \qquad \lambda_i \geq 0$

(2) $z_3(s_3) = c_1 \lambda_1 + c_2 \lambda_2 + \cdots + c_{k+1} \lambda_{k+1}$

under the condition

$$1 = \lambda_1 + \lambda_2 + \cdots + \lambda_{k+1}$$

where λ_i is the activity starting from the origin to the i-th breakpoint

a_i is the value of s_3 at the i-th breakpoint

c_i is the value of z_3 at the i-th breakpoint

$i = 1, \ldots, k$

The next step will be to find the optimal program for the third stage and treat s_2 as a parameter. The matrix form is shown in Fig. 6. We just repeat the procedure of stage 4 for stage 3 and get a broken line convex function $z_2(s_2)$.

We proceed to compute $z_1(s_1)$ by the same device. As the initial resources are known, we are able to solve the optimal program for the first stage activities. From the optimum program of first stage, the value of s_1 is obtained and may be used to determine the optimal program for the second stage activities and the value of s_2, which in turn lead to the solution for the third stage, etc.

The functional equation approach may be extended to two variables such as (s, t), which are shared between successive stages, but the computational work would make the approach tedious to apply.

3. Application of the approach to the case farm

The functional equation approach will now be applied to the above two years program of the case farm in the Case II. The first step in the functional equation approach is to find the optimum plan for the second year with the capital as a variable resource. Using the technique of variable resource programming, the optimum plan is shown as follows:

	Operating capital needed	P_{11}	P_{13}	P_{15}	P_{18}	Net revenue
	NT$	ares	ares	ares	NT$	NT$
λ_1	30,630.61	197.099	282.901	450.000	10,706	53,914.807

In the second step we add the following items to the first year program and form a program as shown in Table 20.

(a) The activity λ_1 with $a_1 = 30,630.61$; $c_1 = 53,914.807$

(b) The additional constraint $\lambda_1 = 1$

The simplex method is applied to solve the problem. The final solution is shown as follows:

$P_2 = 170.825$, $P_8 = 2[illegible]0.000$, $P_9 = 1[illegible],486.000$,

$P_{21} = 29.175$, $P_{23} = 102.059$, $P_{20} = 7[illegible].320$, $\lambda_1 = 0.93373$,

$P_{\lambda} = 0.06627$, $Z-C = 72,364.81$

According to this solution, the optimum plan for the

Table 20. First year programming for the case farm including λ_1

Resource or activity		Resource or activity level P_0	Operating capital$_1$ P_{20}	Spring land$_1$ P_{21}	Fall land$_1$ P_{22}	Spring labor$_1$ P_{23}	Fall labor$_1$ P_{24}	Family living$_1$ P_{25}	Op ca
Operating capital$_1$	P_{20}	17,000NT$	1	0	0	0	0	0	
Spring land$_1$	P_{21}	200 are	0	1	0	0	0	0	
Fall land$_1$	P_{22}	200 are	0	0	1	0	0	0	
Spring labor$_1$	P_{23}	190 days	0	0	0	1	0	0	
Fall labor$_1$	P_{24}	190 days	0	0	0	0	1	0	
Family living$_1$	P_{25}	10,486NT$	0	0	0	0	0	1	
Operating capital$_2$	P_{19}	0NT$	0	0	0	0	0	0	
	P_{λ}	1	0	0	0	0	0	0	
Z - C		0	0	0	0	0	0	0	

amily living[1] P_{25}	Operating capital[2] P_{19}	P_{λ}	λ_1	Spring sweet potatoes[1] P_1	Spring "hu-tze" sweet potatoes[1] P_2	Spring peanut[1] P_3	Jute[1] P_4	Fall sweet potatoes[1] P_5
0	0	0	0	20.67970	21.94210	21.13290	32.61800	16.24990
0	0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	1
0	0	0	0	0.38970	0.51480	0.76320	1.33860	0
0	0	0	0	0	0	0	0	0.32520
1	0	0	0	0	0	0	0	0
0	1	0	30,630.61	-64.37970	-71.63210	-57.26290	-94.07800	-79.33990
0	0	1	1	0	0	0	0	0
0	0	0	-53,914.81	-43.70	-49.69	-36.13	-61.46	-63.09

Jute$_1$ P_4	Fall sweet potatoes$_1$ P_5	2nd rice$_1$ P_6	Fall peanut$_1$ P_7	Ratoon sugar cane$_1$ P_8	Family living$_1$ P_9
32.61800	16.24990	13.82870	19.50130	32.46140	1
1	0	0	0	1	0
0	1	1	1	1	0
1.33860	0	0	0	0.96420	0
0	0.32520	0.59840	0.68590	0.37990	0
0	0	0	0	0	1
4.07800	-79.33990	-81.69870	-69.99130	-127.12140	0
0	0	0	0	0	0
1.46	-63.09	-67.87	-50.49	-94.66	-M

farm in the two-year period is to produce 170.825 ares of spring "hu-tze" sweet potatoes and 200 ares of second rice and to have λ_1 at the level of 0.93373 in the second year, thus yielding the total net revenue of NT$72,364.81 in terms of the present money value during the two-year period.

With the value of λ_1 now known, it is necessary to go back to the solution of the second year program and multiply it through by λ_1 = 0.933, and so obtain the following results:

0.933 (30,630.616 197.099 252.901 450 10,706
53,914.807)

= (28,578.359 183.893 235.957 419.850
9,988.698 50,302.515)

These figures indicate that the case farm should produce 183.893 ares of spring "hu-tze" sweet potatoes, 235.957 ares of jute, 419.850 ares of second rice, spend NT$9,988.698 on family living and obtain the discounted net revenue of NT$50,302.515. The operating capital of NT$28,578.359 required for the above production level would be supplied by the first year's activity. The complete solution for the two year program is as follows:

Year 1 P_2 = 170.825, P_6 = 200.000, P_9 = 10,456.000
P_{21} = 29.175, P_{23} = 102.059, P_{24} = 70.320
Z-C = 22,062.294

Year 2 $P_{11} = 183.893$, $P_{13} = 235.957$, $P_{15} = 419.850$

$P_{18} = 9{,}986.698$, $P_{26} = 30.150$, $P_{27} = 30.150$

$P_{28} = 29.480$, $P_{29} = 188.760$, $Z\text{-}C = 50{,}302.520$

Total net revenue: NT$72,364.81

The final results from using the functional equation approach are the same as using the decomposition algorithm.

V. NONLINEAR PROGRAMMING FOR THE CASE FARM

Certain kinds of nonlinearitics - dis-economies of scale such as decreasing marginal productivity - may be incorporated within a linear programming model. But for economies of scale such as increasing marginal productivity, an attempt to employ linear programming is liable to produce results that are entirely misleading.

Situations with increasing marginal productivity occur widely in agriculture. For example, the real situation of the labor requirement per are on our case farm is not as presented in Table 7. The variable labor charge per are decreases as the number of ares increases. In addition, there is a fixed labor charge. For illustration, consider the spring labor used for jute production, as put forth in Table 21. The items composing the fixed labor charge are:

Table 21. Spring labor used for production of jute of the case farm

	Jute A 1-50 area	Jute B 51-100 ares	Jute C 101-150 ares	Jute D 151-200 ares
Fixed labor charge (days)	5	5	5	5
Variable labor charge (days per are)	1.20	1.08	1.02	0.96

1) transportation to and from the field, 2) preparation of implements, 3) maintenance of a buffalo, etc.

It is assumed that the restriction of resources, real activities, and the input-output coefficients - except for jute - are the same as in year 1 in Table 7. It can be seen, from Table 21, that the use of labor for jute production presents difficulties for the application of the linear programming method. A certain amount of fixed labor charge has to be made and the variable labor charge per are decreases as the number of ares increases for jute production. This is a violation of the basic assumptions of divisibility, continuity and convexity in linear programming. With these peculiar characteristics, the problem should be solved with the "modified simplex method" or the mixed integer programming algorithm.

A. The Modified Simplex Method

The set-up of the present study allows for four jute activities: jute A, 1-50 ares; jute B, 51-100 ares; jute C, 101-150 ares; and jute D, 151-200 ares. It is assumed that the yield, the net revenue, and the use of all resources except labor are proportional to the number of ares. Therefore, all the coefficients - except labor's - for jute B are equal to 51 times the respective coefficients of jute A, and those for jute C and jute D are equal to 101 and 151 times those of jute A respectively. The fixed labor charge for the jute production can be subtracted out of spring labor supply in year 1 of Table 7. The remainder is entered in the P_0 column of the simplex table.

The complete simplex table for the case farm is shown in Table 22. After it was solved by the usual simplex procedure, the optimal solution and final solution matrix were obtained as in Table 23. In the solution, the level of jute D is approximately 126 ares. It should therefore belong to the jute C category.

Since the optimal solution for jute production is 126 ares, and a desirable optimal plan should have the four jute activities mutually exclusive of each other, the most profitable activity should be jute C. We drop activities of jute A, jute B and jute D from Table 22. The remainder of Table 22 is remaximized by the application of the simplex method. The optimum plan for the problem is shown in Table 24.

Since our problem is the decreasing labor cost, the key to the feature lies in the unused spring labor. Because there is 0.83507 units of jute D in the optimal solution of the simplex method, the unused spring labor is 30.92 days. But the 0.83507 units of jute D is equivalent to 126 ares; it is within the range of jute C, therefore, there should be some spring labor artificially unutilized in the program. After remaximization, the level of crop productions is the

Table 22. Simplex table for linear programming of the case farm planning problem

Resource or activity		Resource or activity level P_0	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	Spring peanut P_3	Jute A P_4	Jute B P_5
	$c_j \rightarrow$		43.70	49.69	36.13	61.46	3,134.46
Spring land	d_1	200	1	1	1	1	51
Fall land	d_2	200	0	0	0	0	0
Operating capital	d_3	8,500	20.67970	21.94210	21.13290	32.61800	1,663.518
Spring labor	d_4	185	0.38970	0.51480	0.76820	1.20000	55.080
Fall labor	d_5	190	0	0	0	0	0
Z-C			-43.70	-49.69	-36.13	-61.46	-3,134.46

roblem (disposal activities not shown)

134.46	6,207.46	9,280.46	63.09	67.87	50.49	94.66
ute B P_5	Jute C P_6	Jute D P_7	Fall sweet potatoes P_8	2nd rice P_9	Fall peanut P_{10}	Ratoon sugar cane P_{11}
51	101	151	0	0	0	1
0	0	0	1	1	1	1
563.51800	3,294.41800	4,925.31800	16.24990	13.82870	19.50130	32.46140
55.08000	103.02000	144.96000	0	0	0	0.96420
0	0	0	0.32520	0.59840	0.68590	0.37990
134.46	-6,207.46	-9,280.46	-63.09	-67.87	-50.49	-94.66

Table 23. Optimal solution for the case farm planning problem

Resource or activity		Resource or activity level P_0	Spring land d_1	Fall land d_2	Operating capital d_3	Spring labor d_4	Fall labor d_5	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	
Spring "hu-tze" sweet potatoes	P_2	73.97738	3.05642	1.29601	-0.09371	0	0	1.11831	1	
2nd rice	P_9	200.00000	0	1	0	0	0	0	0	
Spring labor	d_4	30.92255	0.49056	0.63360	-0.04581	1	0	-0.06725	0	
Fall labor	d_5	70.31991	0	-0.59840	0.04327	0	1	0	0	
Jute D	P_7	0.83507	-0.01360	-0.00858	0.00062	0	0	-0.00078	0	-
Z		24,999.76974	25.66	52.64	1.10	0	0	48.33	49.69	4
Z - C		24,999.76974	25.66	52.64	1.10	0	0	4.63	0	1

.ng ;ze" :t ;oes ?	Spring peanut P_3	Jute A P_4	Jute B P_5	Jute C P_6	Jute D P_7	Fall sweet potatoes P_8	2nd rice P_9	Fall peanut P_{10}	Ratoon sugar cane P_{11}
	1.07586	0	0	0	0	-0.22691	0	-0.53163	1.29601
	0	0	0	0	0	1	1	1	1
	0.29050	0.29632	6.43312	4.99686	0	-0.11093	0	-0.25991	0.59412
	0	0	0	0	0	-0.27320	0	0.08751	-0.21850
	-0.00050	0.00662	0.33775	0.66888	1	0.00150	0	0.00352	-0.00196
69	48.82	61.44	3134.47	6270.51	9280.46	70.52	67.87	74.12	114.08
	12.69	0	0	0	0	7.43	0	23.63	19.42

Table 24. Optimum plan for the case farm planning problem

Optimum combination of crops			Disposal resources			Net returns	Limiting resources	
Spring "hu-tze" sweet potatoes	P_2	73.96070 ares	d_4	Spring labor	18.35749 days	NT$11,422.14	d_1	Spring land
Jute C	P_6	1.24802 (126.05 ares)	d_5	Fall labor	70.32011 days		d_2	Fall land
2nd rice	P_{10}	200.00000 ares					d_3	Operating capital

same as before, but the unused spring labor days are reduced to 18.35794 days.

It is clear that we can apply the above modified simplex method to solve non-linear programming problems successfully.

It is worthwhile to note that if the activity of decreasing labor cost is not included in the optimal solution, this problem is a linear programming one; we can solve it with the ordinary linear programming method.

B. The Mixed Integer Programming Algorithm

1. The algorithm

The mixed integer programming algorithm is a method to solve problems of a nonlinear, nonconvex, and combinational character. Economies of scale are an example of such problems which are very difficult to be solved by the linear programming technique.

The algorithm is an extension of the cutting plane technique for the solution of the "pure integer" problem. The cutting plane method was first proposed and demonstrated by Dantzig *et al*. (11). The method

> . . . consists in first solving the linear programming problem without the integer constraints. If the optimum solution happens to satisfy these conditions, all is well. If not, then additional linear inequality constraints (call cutting planes) are added to the system in such a way as to remove the non-admissible extreme point solution and yet retain all admissible solutions (e.g., these having integer values). (9, p. 31)

Markowitz and Manne explored this technique further on problems involving economies of scale and pointed out how it could be applied to solve problems involving nonlinear objective forms (20).

Recently, Gomory presented a method of automatically generating "cutting planes" which permits efficient solution of linear programs in integers (16). This approach has been generalized by Beale (1), by Dantzig (8, 9) and by Land and Doig (18) to cases where some variables are continuous and some are constrained to be integers. More recently, Gomory developed an algorithm for the mixed integer problem (15). This new development makes it possible to solve nonlinear, nonconvex and combinatorial problems by the linear programming approach.

Gomory's method for solving mixed integer problems is outlined as follows (15):

The problem is to maximize an objective function

$$Z = a_{o,o} + \sum_{j=1}^{n} a_{o,j}\ (-t_j)$$

Subject to the inequalities

$$(1) \qquad \sum_{j=1}^{n} a_{i,j}\ t_j \leq a_{i,o}\ , \qquad i = 1, \ldots, m$$

with some specified t_j's should be integers,

where t_j = non-basic variables,

$a_{o,o}$ = coefficient in the 0 columns,

$a_{o,j}$ = coefficient in the objective form

$a_{i,j}$ = input-output coefficients, and

$a_{i,o}$ = coefficients on the right hand side.

If the inequalities in equation 1 are changed into equations in non-negative variables by the addition of m "slack" variables, the results are:

$$(2) \quad X_i = a_{i,o} + \sum_{j=1}^{n} a_{i,j}\,(-t_j) \qquad i = 0, \ldots, m + n$$

where X_i = basic variables,

$a_{i,o}$ = coefficients in the 0-column,

$a_{i,j}$ = the coefficients of the non-basic variables, and

t_j = non-basic variables.

Applying Dantzig's simplex method brings equation 2 into a form which denotes the new coefficients in the equations by primes as follows:

(a) $\quad a'_{i,o} \geqslant 0 \qquad i = 1, \ldots, m + n$

(b) $\quad a'_{o,j} \geqslant 0 \qquad j = 1, \ldots, n$

Condition (a) is obtained by setting all the non-basic variables equal to zero. The values that result for all the variables are non-negative. Condition (b) makes certain that the objective function is maximal. The solution obtained is:

$$X_i = a'_{i,o} \qquad i = 0, \ldots, m + n$$

If this solution does not satisfy the integer requirement, we will be able to make use of equation 3 below to deduce a new inequality that will be satisfied by any integer solution.

$$X_i = a'_{i,o} + \sum a'_{i,j}\ (-t_j) \tag{3}$$

where X_i is an integer variable, $a'_{i,o}$ is a non-integer, $a'_{i,j}$ is a current coefficient of a non-basic variable, and t_j is the current set of non-basic variables. Through equation 3 the following equation is developed:

$$S_i = -f'_{i,o} - \sum f^*_{i,j}\ (-t_j) \tag{4}$$

where S_i is a basic variable, the $f'_{i,o} = a'_{i,o} - n_{i,o}$; $n_{i,o}$ the largest integer $\leq a'_{i,o}$, and the $f^*_{i,j}$, all non-negative, are given by the following formulae:

$$f^*_{i,j} = \begin{cases} a'_{i,j} & \text{if } a'_{i,j} \geq 0 \text{ and } t_j \text{ non-integer variable} \\ \dfrac{f'_{i,o}}{1-f'_{i,o}}\ (-a'_{i,j}) & \text{if } a'_{i,j} < 0 \text{ and } t_j \text{ non-integer variable} \\ f'_{i,j} & \text{if } f'_{i,j} \leq f'_{i,o} \text{ and } t_j \text{ integer variables} \\ \dfrac{f'_{i,j}}{1-f'_{i,o}}\ (f'_{i,j} - 1) & \text{if } f'_{i,j} > f'_{i,o} \text{ and } t_j \text{ integer variable} \end{cases} \tag{5}$$

where $f'_{i,j} = a'_{i,j} - n_{i,j}$; $n_{i,j}$ the largest integer $\leq a'_{i,j}$. Equation 4 is the additional restrictions that are employed in solving the integer programming problem and the problem is remaximized. This process is repeated until an integer solution is obtained.

The algorithm for solving an integer programming problem can be summarized as follows:

(a) Solve the original problem by the simplex method.

(b) If the solution is non-integer add any additional constraints equation 4.

(c) Repeat this process until an integer maximum is obtained.

2. Application of mixed integer programming to the case farm

The above case of the decreasing labor cost in jute planning can also be handled by the mixed integer programming technique, with a restriction imposed on each of the jute activities P_i, such that

$$P_i = 0 \quad \text{or} \quad P_i \geqslant 1$$

The conditions of this restriction are: (a) there is no value for P_i between zero and one; and (b) P_i is continuous if it is greater than one. The former prevents the program from presenting any of the less efficient jute activities as a fraction of the most efficient one in terms of labor use; and the latter recognizes the fact of a fraction of the activity unit which is the lower bound of the respective size of jute production.

The upper bounds for the four jute activities are: $U_a = 50$, $U_b = 100/51 = 1.96078$, $U_c = 150/101 = 1.48515$, and $U_d = 200/151 = 0.75500$. With the fixed labor charge a = 5 hours, we have $a/U_a = 0.10000$, $a/U_b = 2.55000$, $a/U_c = 3.36666$, and $a/U_d = 6.62252$. The simplex setup for the four jute activities and the fixed labor charge is shown in Table 25.

Table 25. Simplex table for linear programming of the case farm planning problem (dis

Resource or activity		Resource or activity level P_0	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	Spring peanut P_3	Jute A P_4	Jute B P_5	
		$c_j \rightarrow$	43.70	49.69	36.13	61.46	3,134.46	6
Spring land	d_1	200	1	1	1	1	51	
Fall land	d_2	200	0	0	0	0	0	
Operating capital	d_3	8,500	20.67970	21.94210	21.13290	32.61800	1,663.51800	3
Spring labor	d_4	190	0.38970	0.51480	0.76820	1.20000	55.08000	
Fall labor	d_5	190	0	0	0	0	0	
Fixed labor charge	d_6	0	0	0	0	0.10000	2.55000	
Z-C		0	-43.70	-49.69	-36.13	-61.46	-3,134.46	-6,

oblem (disposal activities not shown)

34.46	6,207.46	9,280.46	0	63.09	67.87	50.49	94.66
te B	Jute C P_6	Jute D P_7	Fixed labor charge P_8	Fall sweet potatoes P_9	2nd rice P_{10}	Fall peanut P_{11}	Ratoon sugar cane P_{12}
1	101	151	0	0	0	0	1
0	0	0	0	1	1	1	1
3.51800	3,294.41800	4,925.31800	0	16.24990	13.82870	19.50130	32.46140
5.08000	103.02000	144.96000	5.0	0	0	0	0.96420
0	0	0	0	0.32520	0.59840	0.68590	0.37990
2.55000	3.36666	6.62252	-5.0	0	0	0	0
.46	-6,207.46	-9,280.46	0	-63.09	-67.87	-50.49	-94.66

The complete simplex table for the case farm planning problem is shown in Table 25. After it was solved by the simplex method, the continuous optimal solution and final solution matrix were obtained, as shown in Table 26, for the use of the mixed integer programming. In the continuous solution, the level of jute D is equivalent to 126 ares. It should belong to jute C category. Also, the fixed labor charge is an undesirable fraction - 1.10557.

These two fractional parts of the activities in the continuous solution will be removed by the mixed integer programming technique. The fraction for jute D is removed first. The row corresponding to jute D in the final solution matrix is the sixth row in Table 26. Expressed in the form of equation 3 as follows:

$$\begin{aligned} P_7 = {} & 0.83507 - 0.01360\ (-d_1) - 0.00858\ (-d_2) \\ & + 0.00062\ (-d_3) - 0.00078\ (-P_1) - 0.00050\ (-P_3) \\ & + 0.00662\ (-P_4) + 0.33775\ (-P_5) + 0.66888\ (-P_6) \\ & + 0.00150\ (-P_7) - 0.00352\ (-P_{11}) - 0.00196\ (-P_{12}) \end{aligned}$$

In the above equation, the non-basic variables P_4, P_5 and P_6, which are jute A, jute B and jute C respectively, are treated as integer variables before their value greater than one. The following new constraint is formed by the application formulae of equation 5.

Table 26. Mixed integer programming for the case farm planning problem with the

Resource or activity		Resource or activity level P_0	Spring land d_1	Fall land d_2	Operating capital d_3	Spring labor d_4	Fall labor d_5	Fixed labor charge d_6
Fixed labor charge	P_8	1.10557	-0.01803	-0.001137	0.00082	0	0	-0.20000
Spring "hu-tze" sweet potatoes	P_2	73.97738	3.05642	1.29601	-0.09371	0	0	0
2nd rice	P_{10}	200.00000	0	1	0	0	0	0
Spring labor	d_4	60.39470	0.49056	0.63360	-0.04581	1	0	1
Fall labor	d_5	105.31991	0	-0.59840	0.04327	0	1	0
Jute D	P_7	0.83507	-0.01360	-0.00858	0.00062	0	0	0
	S_7	0.83507	0.06886	0.04344	0.00062	0	0	0
Z - C		24,999.76974	25.66	52.64	1.10	0	0	0

th the additional constraint corresponding to the jute D

ixed abor arge d_6	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	Spring peanut P_3	Jute A P_4	Jute B P_5	Jute C P_6	Jute D P_7	Fixed labor charge P_8	Fa swe pota P
20000	-0.00104	0	-0.00067	-0.01123	-0.06265	0.21259	0	1	0.C
	1.11831	1	1.07586	0	0	0	0	0	-0.2
	0	0	0	0	0	0	0	0	1
	-0.06725	0	0.29050	0.29632	6.43312	4.99686	0	0	-0.1
	0	0	0	0	0	0	0	0	-0.2
	-0.00078	0	-0.00050	0.00662	0.33775	0.66888	1	0	0.0
	0.00395	0	0.00253	0.00662	0.33775	0.66888	0	0	0.0
	4.63	0	12.69	0	0	0	0	0	7.4

xed bor arge P_8	Fall sweet potatoes P_9	2nd rice P_{10}	Fall peanut P_{11}	Ratoon sugar cane P_{12}	S_7	S_7'
	0.00199	0	0.00466	-0.00264	0	0
	-0.22691	0	-0.53163	1.29601	0	0
	1	1	1	1	0	0
	-0.11093	0	-0.25991	0.59412	0	0
	-0.27320	0	0.08751	-0.21850	0	0
	0.00150	0	0.00352	-0.00196	0	0
	0.00150	0	0.00352	0.00992	-1	1
	7.43	0	23.63	19.42	0	10,000

$$S_7 = -0.83507 - 0.06886\ (-d_1) - 0.04344\ (-d_2) - 0.00062\ (-d_3) - 0.00395\ (-P_1) - 0.00253\ (-P_3) - 0.00662\ (-P_4) - 0.33775\ (-P_5) - 0.66888\ (-P_6) - 0.00150\ (-P_9) - 0.00352\ (-P_{11}) - 0.00992\ (-P_{12})$$

This row is multiplied through by -1 in order to avoid negative figures in the right hand side column and added to the final solution matrix for remaximization. This operation set -1 in the cell of the disposal activity of S_7 and made it necessary to add a positive artificial disposal activity S_7', with penalty 10,000 attached, to the final solution matrix. Table 26 is the completed form for the starting of the mixed integer programming procedure. The simplex method is applied for remaximization; its optimal solution is shown from rows 1-7 in Table 27.

The mixed integer optimal solution shows that the fixed labor charge is still a fraction - 0.84016 - hence a second constraint, corresponding to the fixed labor charge, is added to the final mixed integer solution matrix. The complete simplex table for the second remaximization is shown in Table 27. The mixed integer optimal solution is obtained as shown in rows 1-8 in Table 28, but jute C still has a fraction - 0.73733 - thus a third constraint corresponding to the jute C has to be added to the final mixed integer solution matrix of Table 28. The complete simplex table for the third remaximization is shown in Table 28.

Table 27. Mixed integer programming for the case farm planning p

Resource or activity		Resource or activity level P_0	Spring land d_1	Fall land d_2	Operating capital d_3	Spring labor d_4
Fixed labor charge	P_8	0.84016	-0.03992	-0.02518	0.00062	0
Spring "hu-tze" sweet potatoes	P_2	73.97738	3.05642	1.29601	-0.09371	0
2nd rice	P_{10}	200.00000	0	1	0	0
Spring labor	d_4	54.15632	-0.02387	0.30910	-0.05046	1
Fall labor	d_5	105.31991	0	-0.59840	0.04327	0
Jute D	P_7	0	-0.08246	-0.05202	0	0
Jute C	P_6	1.24846	0.10295	0.06494	0.00093	0
	S_8	0.84016	0.20983	0.13235	0.00062	0
Z - C		24,999.76974	25.66	52.64	1.10	0

...ing problem with the additional constraint corresponding to the fixed lab...

Spring labor d_4	Fall labor d_5	Fixed labor charge d_6	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	Spring peanut P_3	Jute A P_4	Jute B P_5	Jute C P_6
0	0	-0.20000	-0.00230	0	-0.00147	-0.01333	-0.17000	0
0	0	0	1.11831	1	1.07586	0	0	0
0	0	0	0	0	0	0	0	0
1	0	1	-0.09678	0	0.27161	0.24605	3.90996	0
0	1	0	0	0	0	0	0	0
0	0	0	-0.00473	0	-0.00303	0	0	0
0	0	0	0.00591	0	-0.00378	0.00990	0.50495	1
0	0	0.80000	0.01209	0	0.00773	0.07007	0.83000	0
0	0	0	4.63	0	12.69	0	0	0

the fixed labor charge

e B P_5	Jute C P_6	Jute D P_7	Fixed labor charge P_8	Fall sweet potatoes P_9	2nd rice P_{10}	Fall peanut P_{11}	Ratoon sugar cane P_{12}	S_8	$S_8^{'}$
7000	0	0	1	0.00151	0	0.00354	-0.00579	0	0
	0	0	0	-0.22691	0	-0.53163	1.29601	0	0
	0	0	0	1	1	1	1	0	0
996	0	0	0	-0.12212	0	-0.28619	0.52002	0	0
	0	0	0	-0.27320	0	0.08751	-0.21850	0	0
	0	1	0	0	0	0	-0.01188	0	0
495	1	0	0	0.00224	0	0.00526	0.01483	0	0
000	0	0	0	0.00151	0	0.00354	0.03043	-1	1
	0	0	0	7.43	0	23.63	19.42	0	10,000

Table 28. Mixed integer programming for the case farm planning with the additional

Resource or activity		Resource or activity level P_0	Spring land d_1	Fall land d_2	Operating capital d_3	Spring labor d_4	Fall labor d_5	Fixed labor charge d_6	Sp sw pot
Fixed labor charge	P_8	1.01224	0.00306	0.00193	0.00075	0	0	-0.03614	0.(
Spring "hu-tze" sweet potatoes	P_2	73.97738	3.05642	1.29601	-0.09371	0	0	0	1.1
2nd rice	P_{10}	200.00000	0	1	0	0	0	0	0
Spring labor	d_4	50.19850	-1.01235	-0.31438	-0.05335	1	0	-2.76865	-0.1
Fall labor	d_5	105.31991	0	-0.59840	0.04327	0	1	0	0
Jute D	P_7	0	-0.08246	-0.05202	0	0	0	0	-0.0
Jute C	P_6	0.73733	-0.02471	-0.01558	0.00556	0	0	-0.48670	-0.0
Jute B	P_5	1.01224	0.25281	0.15946	0.00074	0	0	0.96386	0.0
	S_6	0.73733	0.06936	0.04373	0.00056	0	0	0.51330	0.0
Z - C		24,999.76974	25.66	52.64	1.10	0	0	0	4.6

ıdditional constraint corresponding to the jute C

xed .bor arge 6	Spring sweet potatoes P_1	Spring "hu-tze" sweet potatoes P_2	Spring peanut P_3	Jute A P_4	Jute B P_5	Jute C P_6	Jute D P_7	Fixed labor charge P_8	Fa swe pota P
03614	0.00018	0	0.00011	0.00102	0	0	0	1	0.0
	1.11831	1	1.07586	0	0	0	0	0	-0.2
	0	0	0	0	0	0	0	0	1
76865	-0.15375	0	0.23521	-0.08323	0	0	0	0	-0.1
	0	0	0	0	0	0	0	0	-0.27
	-0.00473	0	-0.00303	0	0	0	1	0	0.
8670	-0.00145	0	-0.00092	-0.03273	0	1	0	0	0.00
6386	0.01457	0	0.00931	0.08442	1	0	0	0	0.00
1330	0.00407	0	0.00258	0.09188	0	0	0	0	0.00
	4.63	0	12.69	0	0	0	0	0	7.43

C	Jute D P_7	Fixed labor charge P_8	Fall sweet potatoes P_9	2nd rice P_{10}	Fall peanut P_{11}	Ratoon sugar cane P_{12}	S_6	S_6'
	0	1	0.00182	0	0.00427	0.00044	0	0
	0	0	-0.22691	0	-0.53163	1.29601	0	0
	0	0	1	1	1	1	0	0
	0	0	-0.12924	0	-0.30289	0.37668	0	0
	0	0	-0.27320	0	0.08751	-0.21850	0	0
	1	0	0	0	0	-0.01188	0	0
	0	0	0.00132	0	0.00310	-0.00368	0	0
	0	0	0.00182	0	0.00427	0.03666	0	0
	0	0	0.00132	0	0.00310	0.01033	-1	1
	0	0	7.43	0	23.63	19.42	0	10,000

This optimal solution is shown in Table 29, together with the continuous optimal solution and the optimal solution of the modified simplex method, in order to make a comparison.

It is seen in Table 29 that the optimal solutions from the three different methods for the case farm planning problem are identical except for fixed labor charge and unused spring labor in the continuous solution. Since there is a fraction for fixed labor charge and some spring labor artifically unused in the continuous solution, the results of its fixed labor charge and unused spring labor are different from the correct answers as listed in the modified simplex solution or mixed integer solution.

This integer programming algorithm has been applied in another study (7).

Table 29. Optimal solutions for the case farm planning problem

Item		Unit	Modified simplex solution	Continuous solution	Mixed integer solution
Net profit		NT$		24,999.77	24,999.70
Spring "hu-tze" sweet potatoes	P_2	are	73.96070	73.97738	73.97738
Jute:					
Jute A	P_4	unit	--	--	--
Jute B	P_5	unit	--	--	--
Jute C	P_6	unit	1.24802 (126.05 ares)	--	1.24846 (126.09446 ares)
Jute D	P_7	unit		0.83507 (126.09 ares)	
2nd rice	P_{10}	are	200.00000	200.00000	200.00000
Fixed labor charge	P_8	day	5	1.10557 (5.53 days)	1 (5 days)
Unused spring labor	d_4	day	18.35749	25.39470	18.35706
Unused fall labor	d_5	day	70.32011	70.31991	70.31991

VI. INTERPRETATION OF FINDINGS

The foregoing sections have presented the results obtained in the analysis. This section will interpret the findings.

A. Dynamic Linear Programming

The optimum five-year plans for the case farm are given in Tables 8, 10, 12 and 14. Each of them is an optimum plan for five years. The optimum plan for any one year depends on (a) the optimum in other years, (b) the availability of and returns on operating capital in other years, and (c) the need for household consumption at different points in time.

1. Situation I

In the plan for Situation I (Table 8), both spring and fall land and spring labor - except for year 1 - are the limitational resources for the case farm. Therefore, these resources should be used to produce the most profitable crop. Since spring "hu-tze" sweet potatoes and jute, and second rice are the more profitable crops in spring and fall respectively for the case farm, they are grown on the farm during the years 2, 3 and 4. In year 1, it is due to the limitation of operating capital that no jute which is consuming operating capital one is produced. In year 5, because the discounted net revenue of fall sweet potatoes is higher than second rice, the latter is substituted by the former. Second rice is produced from years 1 through 5. Spring "hu-tze" sweet potatoes and jute are produced from years 2 to 5. Jute requires more spring labor than other spring crops; its producing area is increased as spring labor increases each year. Also, the number of ares of jute increasing is the same as the number of ares of spring "hu-tze" sweet potatoes decreasing. For instance, 88.27448 ares of spring "hu-tze" sweet potatoes and 111.72552 ares of jute are produced in year 2, but 63.99688 ares of spring "hu-tze" sweet potatoes and 136.00312 ares of jute are produced as spring labor increases five days in year 3. As production is limited by the scarce resources of spring and fall land and spring labor, there is operating capital and fall labor not used in years 2 through 5. These disposal resources are not transferred to the supplies of these resources for the following year. Because the supplies of land and labor are fixed, the total discounted net returns over the five-year period are NT$113,776.48. This is the lowest of all the situations studied.

2. Situation II

It was stated that for Situation I spring labor is a limiting resource. Since there is no difficulty in hiring spring labor for the prevailing wage rate in the region, it is allowed to hire spring labor from years 1 to 5. In the plan for Situation II (Table 10), spring and fall land are the limiting resources. The case farm should make optimum utilization of its scarce resources of land. Since the shadow prices of spring labor in years 1, 4 and 5 are less than the corresponding cost of hiring spring labor, there is no spring labor hired for these years. The optimum crop combination for these years is the same as under Situation I. In year 2, there are 72.72 days of spring labor hired, all 200 ares of land are utilized to produce jute and second rice in spring and fall respectively. In year 3, 52.72 days of spring labor are hired. The optimum combination of crops is the same as in year 2. It is apparent that if the case farm has unlimited spring labor, it will be profitable to produce jute on all the spring land. Over the 5-year period, the total discounted net returns are NT$113,987.51. This is only NT$211.03 more than under Situation I. This indicates that the activity of hiring spring labor does not increase net returns substantially for the case farm.

3. Situation III

It is seen that land is the most limiting resource under Situations I and II. If the case farm is enlarged by purchasing or renting additional land, it will maximize its profits. It is assumed that the case farm could rent not more than one hectare of land with the other restrictions remaining unchanged. In the plan for Situation III (Table 12), both spring and fall land, and spring labor are still the limiting resources in years 2 through 5. Because there is no land rented in years 1 and 2, the optimum plan for these years is the same as under Situation I. One hectare of land is rented in year 3. The optimum combination of crops is 226.49 ares of spring "hu-tze" sweet potatoes, 73.51 ares of jute and 300 ares of second rice. Since one hectare of land is also rented in years 4 and 5, there are three hectares of land available for production of spring and fall crops. All 300 ares of fall land are used to produce second rice in these two years. It is due to the supply of spring labor in year 5 being more than year 4 that 220.41848 ares of spring "hu-tze" sweet potatoes and 79.58152 ares of jute, and 214.34908 ares of spring "hu-tze" sweet potatoes and 85.65092 ares of jute are produced in years 4 and 5 respectively. Over the five-year period, the total discounted net returns are NT$127,760.26. This exceeds the NT$13,983 under Situation I. It becomes obvious that the renting of land is the most effective way to increase net returns for the farm.

4. Situation IV

This situation is the combination of Situations I, II and III. In the plan for Situation IV (Table 14), spring and fall land are still the limiting resources except for year 1. Operating capital is also a limiting resource in years 1 and

3. The optimum plan for year 1 is the same as under Situations I, II or III. There are 72.72052 days of spring labor hired in year 2. The optimum plan is the same as under Situation II. In years 3, 118.00 days of hiring spring labor and 100 ares of renting land are introduced in the plan. The optimum combination of crops is 83.25 ares of spring "hu-tze" sweet potatoes, 216.75 ares of jute and 300.00 ares of second rice. Since the shadow prices of spring labor in years 4 and 5 are less than the corresponding cost of hiring spring labor, no spring labor is hired. The optimum combination of crops in years 4 and 5 is the same as under Situation III. Over the five-year period, the total discounted net returns are NT$128,013.82. This exceeds NT$14,237.34 under Situation I and is the highest one of all situations studied.

The results of the above optimum combination is the real picture of farm plans for the single-cropping paddy farms in Chaiyi, Taiwan. Farmers with limited operating capital, as in year 1, under the different situations of the case farm, usually produce spring "hu-tze" sweet potatoes in spring and second rice in fall for the following reasons:

(1) Spring "hu-tze" sweet potatoes are planted in late October before the harvest of the second rice in order to save time and make intensive use of land.

(2) The price of spring "hu-tze" sweet potatoes is usually higher than spring sweet potatoes.

(3) Spring "hu-tze" sweet potatoes are the main source of feed for hog production.

(4) Rice is the main source of food for home consumption.

(5) Both rice and spring "hu-tze" sweet potatoes are stable crops.

On the other hand, rich farmers usually diversify their spring crop production by producing spring "hu-tze" sweet potatoes and jute in the spring. Second rice is then produced in the fall. This is shown for years 2 through 5 of the different situation, which indicates that operating capital is not a limited resource for the case farm. Jute is a cash crop and requires more capital and labor than other spring crops. Due to the shortage of spring labor, most farms produce spring "hu-tze" sweet potatoes and jute on the farm in the spring. Rice is the most profitable crop in the fall. Virtually every farm produces rice in order to maximize profits.

Land is the most limited resource in the region. However, the enlargement of the land area is the most effective way to increase net returns for the case farm. There are only 883,466 hectares of cultivated land in Taiwan, 535,674 hectares of which are paddy fields and 349,792 hectares of upland farms. The number of farm families in Taiwan is recorded as 769,925. This gives an agricultural population of 4,880,901 which makes up 48.62 per cent of the total population in Taiwan. Under such a farm land and population ratio, it is very difficult for farmers to increase the supply of the land resource. Therefore, our assumption for the case farm in Situations III and IV, which allowed the renting of not more than one hectare of land in years 3 through 5 is not too realistic.

Under situations I and II, we found negative discounted net income in years 4 and 5. We might ask ourselves whether we could remove such negative income if we extended the model beyond five years. Since both spring and fall land were limited to two hectares which limited the crop production, and the family living costs and fixed expenditures were increased year after year, we could not remove them even though we extended it beyond five years.

As to Situations III and IV, we also found negative discounted net income in year 3, but it was an artificial figure, because we charged the rent of years 4 and 5 in year 3. If we added these rents to the discounted net returns in year 3, then the discounted net income in year 3 would be NT$4,182.59 and NT$5,260.19 under Situations III and IV respectively. It is evident that we can remove the negative income by enlargement of the farm size or by reduction of the family living costs and fixed expenditures.

It might seem unreasonable not to utilize disposal capital for production under different situations. However, the profit function to be maximized in this dynamic linear programming solution concerns a multi-year period; and crop production is interrelated over all years of the whole period. For instance, under Situation I profits are maximized for the 5-year period by investing only NT$32,803.18 in year 3; NT$9,022.52 is available, but is not invested.

B. Nonlinear Programming

The modified simplex method or the mixed integer programming algorithm has been applied to solve the nonlinear programming problem successfully for the case farm. In the example, spring labor for production of jute decreases as the number of ares increases. There is also a fixed labor charge. Such increasing marginal productivity of labor or fixed capital occurs widely in the production of crops and livestock in Taiwan.

In Tables 23 and 24 of the previous section, the optimal plan for the level of crop productions are the same, but the unused spring labor days are different. The solution of the ordinally simplex method presents 30.92 days of unused spring labor, but the unused spring labor days are reduced to 18.36 days by the modified simplex method. It indicated that there are 12.56 days of spring labor artificially unutilized in the program by the simplex method. It is easy to see that if such economies of scale problems are solved by the linear programming method, the result will be erroneous as seen in the case farm example.

C. Use in Extension

The potential use of applying dynamic and nonlinear programming techniques to the individual farm and home unit depends on data, trained help and mechanical aid availability and types of situations to be analyzed. Since the majority of farms in Taiwan do not keep complete farm records, the data required for linear programming is seldom available. In addition, input-output coefficients for use in linear or nonlinear programming must be obtained on an enterprise or activity basis. Accordingly, conventional farm business records are not easily adapted to a programming analysis. Also, there are no complete home accounts available at most farm families. However, we do have some leading farmers in the different rural communities who keep farm records as well as home accounts. For the purpose of demonstration, the extension specialists of farm management and home economics should work with these farmers to get data for their farm and home programming. Since there is no electronic computer available, the separation method is recommended to be used. The modified simplex method is applied to solve nonlinear programming problems.

The question to be answered is whether or not the optimum plan provides guidance in farm organization and home management for the farmers in Taiwan. It can be used as a farm organization and home management model for farmers in the region, if their technology of production, soil, climate, topography, economic conditions, the amount of resources, price, fixed expenditures and home consumption are similar to the case farm. It is especially helpful to young or beginning farmers. In the real world, no two farms exist under exactly the same conditions as mentioned above. Therefore, farmers may necessarily make some modifications of the optimum farm plan in order to meet their own farm situations.

It has been indicated in the previous section that enlargement of the land area is the most effective way to increase net returns for the case farm. However, we also know that it is very difficult for farmers to increase the supply of the land resource due to limitation of land in Taiwan. Since our farm population is about 49 per cent of the total population, it would be best to transfer farm people who operate very small farms to engage in non-farm occupations in order to increase land supply to the existing larger farms.

VII. SUMMARY

The preceding study developed a sequence of yearly plans which provided optimum five-year farm and home programs for a case farm under four situations. The case farm is owner-operated and is located in Chaiyi, Taiwan. Its land is classified as single-cropping paddy land. In each of the five-year plans, family living was considered to be an "exogenous" activity because an exact operating capital allowance for this activity must be met each year. Therefore, family living competes with farm production in the use of available operating capital. In each optimum plan, operating capital generated from crop production in any one year was used for farm production and home consumption for the following year. Dynamic and nonlinear programming were used to obtain the optimum farm and home plans.

The currently available resources of the case farm were two hectares of land, NT$17,000 of operating capital, 190 days of spring and fall labor. Spring planted crops, fall planted crops and an annually-planted crop were to be grown within the year on the case farm. The prices and yields of each crop, and fixed farm expenditures, for the five years were projected. The discounted net revenues were used for programming. The family living costs were categorized and estimated by the farm family.

The four situations studied were the following: Situation

I - land and labor were assumed to be fixed on the case farm; Situation II - the same as Situation I, except spring labor was assumed to be hired; Situation III - the same as Situation I, except land was assumed to be rented; Situation IV - spring labor and land were assumed to be hired and rented, respectively, with the amount of other resources as they were in Situation I.

One of the objectives of this study was to determine optimum farm and home plans for the case farm over a period of five years under the different situations.

Four dynamic linear programming methods were used for solving dynamic problems of the case farm. These methods were the modified simplex method, the separation method, the decomposition algorithm and the functional equation approach. The separation method was developed by this study.

In the optimum plans fall land was used for second rice production except for year 5 and spring land was used for spring "hu-tze" sweet potatoes and jute except for year 1 over the five-year period. In year 5, because the discounted net revenue of fall sweet potatoes was higher than second rice, the latter was substituted by the former. It was due to the limitation of operating capital that there was no jute produced in year 1. Since spring "hu-tze" sweet potatoes competed directly with jute for the use of spring land, the number of ares of jute increasing was the same as the number of ares of spring "hu-tze" sweet potatoes decreasing.

Both spring and fall land were the most limiting resources, except for year 1, under all situations. Spring labor is also the limiting resource in years 2-5 under Situations I and III. The total discounted net returns for the five-year period were highest when spring labor and land were assumed to be hired and rented, respectively. Returns were lowest when neither hiring spring labor nor renting land were included.

The real situation of the labor requirement per are of the different crops for the case farm was that the variable labor charge per are decreased as the number of ares increased and that there was also a fixed labor charge. Economies of scale problems were solved by the modified simplex method and the mixed integer programming algorithm of nonlinear approach for the jute production for the case farm. The modified simplex method was developed and was used to determine an optimum plan for the case farm under conditions of increasing marginal productivity for spring labor.

Important points derived from the study are the following: (1) The dynamic and nonlinear programming problems of the case farm were solved successfully by the different methods as indicated above. (2) Large scale dynamic problems can be easily solved without an electronic computer by the separation method. (3) The optimum combination of crops is spring "hu-tze" sweet potatoes, jute and second rice for the case farm. It reflects the real picture of farm production on the single-cropping paddy farms in Chaiyi, Taiwan. (4) Since land is the most limiting resource of the case farm, the enlargement of the land area is the most effective way to increase net returns for the farm. (5) Optimum farm and home plans can be used as a farm production and home consumption model for farmers in Taiwan, if their technology of production, soil, climate, topography, the amount of resources, economic conditions, price, fixed expenditures and family living costs are similar to the case farm. Otherwise, they should make some modifications of the optimum farm and home plans in order to meet their own situations. (6) For the purpose of using such advanced programming techniques to farm and home planning in Taiwan, the following conditions must be met: (a) complete farm and home records should be designed and kept; (b) extension specialists of farm management and home economics should be trained; (c) electronic computers should be available.

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